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Design of H ∞ Predictive Controller for Networked Control System

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Abstract: For a class of network control systems with both data packet dropout and network communication delay problems, a new robust model predictive control method with compensation function is proposed. Considering that the system has interference problems, in the two cases of long-delay and short-delay, the packet loss problem is established as a Bernoulli sequence, and then a discrete NCS model based on the state observer is obtained. The state observer in the model can deal with the data packet dropout compensate and predict the state of the long-delay problem. Through linear matrix inequality and Lyapunov method, the controller is designed to obtain sufficient conditions for the closed-loop system to be exponentially stable and meet the specified performance indicators. Finally, compared with the method without any compensation measures, the method in this paper can get better control effect.

Keywords: Networked control system; state observer; compensate; H ∞ ; Bernoulli sequence

1. Introduction

The network control system (NCS) is a closed-loop feedback control system formed by connecting various components (sensors, controllers, and actuators) distributed in different regions by means of network technology. As shown in Figure 1, compared to the traditional point-to-point control system, this distributed control system has the advantages of less wiring, low cost of setting up the system, high flexibility and high efficiency, simpler system expansion and maintenance, and remote control.Because of this, network control systems can be widely used in aerospace, factory automation, robotics, remote fault diagnosis and other fields [1-3].

At the same time, the addition of the communication network also complicates the analysis and design of the control system. The limited network bandwidth makes data packet dropout and network communication delay often occur in the network control system. Literature [4] describes the stability study of state feedback network control systems in the presence of network communication delay and data packet dropout. In actual network control system applications, the state of some systems is often unmeasurable. Literature [5-9] designed a state observer based on output feedback. The control system only considered the delay problem, and did not compensate for the impact of the data packet dropout problem. In [10], considering the simultaneous existence of network communication delay and data packet dropout, a state observer is used to compensate the system, and a better control effect is obtained. The problem of data packet dropout in the actual field is a random process, so it is very necessary to establish a random model of the network control system. Literature [11-12] regards the problem of data packet dropout as a random problem, and describes it as a random effort. Sequences and random processes based on Markov. At the same time, the actual engineering systems are all disturbing, and many current articles are studying pure linear systems. In [13] considering the data packet dropout and long network communication delay at the same time, the controller design with disturbing network control system is carried out. Literature [14] compensates the control signal according to the operating parameters of the random long-delay system, so as to achieve the stable control of the random long-delay system. Literature [15] studied the problem of nonlinear networked quantitative control with network communication delay. Literature [16] describes the application of the truetime toolkit in the network control system. The simulation of numerical examples in the article can visually see the effect of the control system.

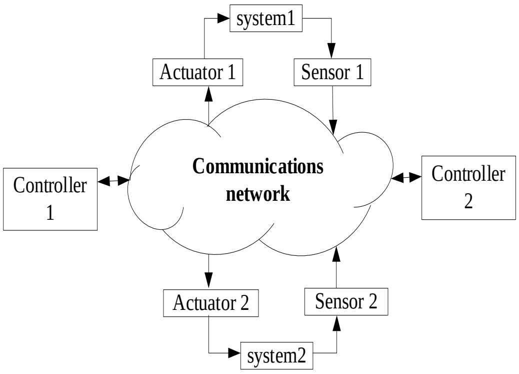

Figure 1: The structure of the network control system.

In this paper, the data packet dropout problem is established as a Bernoulli model, considering the short-delay and long-delay conditions respectively. For the state unobservable problem, a linear system model based on the state observer is constructed to compensate for the data packet dropout and predict the state. Using Lyapunov's theorem and linear matrix inequality and other methods, a controller design method that satisfies the specified performance index is given, so that the closed-loop system is exponentially stable and can withstand certain external disturbances.

2. System Modeling

The network control system structure considered in this article is shown in Figure 2.

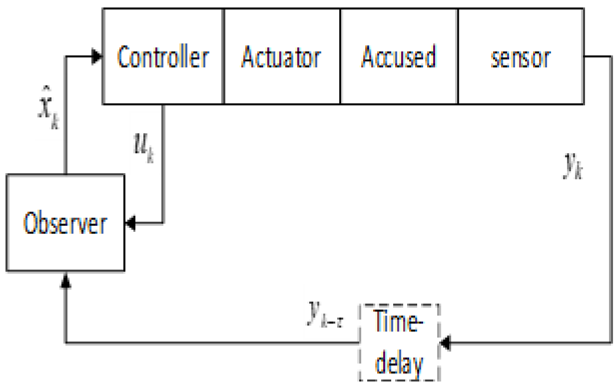

Figure 2: Single-sided random delay network control system structure.

Suppose the state space of the continuous controlled object is described as

$$\begin{cases} \dot{x}(t) = Ax(t) + BU(t) + MG(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state of the controlled object, $u(t) \in R^m$ is the input of the controlled object, $y(t) \in R^p$ is the measured output of the controlled object, $G(t) \in R^p$ is interference, and $G(t) \in L_2[0,\infty)$,and A, B, C, M are constant matrices with corresponding dimensions.

Assume that $(m-1)T < \tau \le mT(m = 1,2,\cdots,M)$, let $\tau' = \tau - (m-1)T$, the discrete time model of equation (1) can be obtained as

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma_1 u_{k-m+1} + \Gamma_2 u_{k-m} + NG_k \\ y_k = Cx_k \\ z_k = Cx_k + NG_k \end{cases} \tag{2}$$

Where $\Phi = e^{AT}$,$\Gamma_1 = \int_0^{T-\tau'} e^{AT} Bdt$,$\Gamma_2 = \int_{T-\tau'}^{T} e^{AT} Bdt$,$N = \int_0^T Mdt$.

The controller is $u_k = Kx_k$. Due to the packet loss phenomenon in the actual system, the controller can be described as $u_k = \alpha_k Kx_k$, $\{\alpha_k\}$ represents the data packet dropout situation at time, $P\{\alpha_k = 1\} = \alpha$, $P\{\alpha_k = 0\} = 1-\alpha$, $0 \le \alpha \le 1$, where α are known constants, and P is the value random Bernoulli sequence of 0 and 1.

2.1. Short network communication delay

When m=1, a short-delay event occurs. This event is marked as E_1, and the probability of occurrence is r_1. When the network communication delay is less than one sampling period, that is, $\tau = \frac{T}{2} + \tau_1 \le T$, where $-\frac{T}{2} \le \tau_1 \le \frac{T}{2}$, Γ_1, Γ_2 can be written in the following form.

$$\Gamma_1 = \int_0^{T-\tau} e^{At} dtB = \int_0^{T-T/2-\tau_1} e^{At} dtB = \int_0^{T/2} e^{At} Bdt + e^{A(T/2)} \int_0^{-\tau_1} e^{At} dtB$$

$$\Gamma_2 = \int_{T-\tau}^{T} e^{At} dtB = \int_{T-T/2-\tau_1}^{T} e^{At} dtB = \int_{T/2}^{T} e^{At} Bdt - e^{A(T/2)} \int_0^{-\tau_1} e^{At} dtB$$

Let $D_1 = \int_0^{T/2} e^{At} Bdt$, $D_2 = \int_{T/2}^{T} e^{At} Bdt$, $\overline{F}(\tau_1) = \int_0^{\tau_1} e^{At} dt$, $E = \delta e^{AT/2}$, $F(\tau_1) = \delta^{-1} \overline{F}(\tau_1)$

$F^T(\tau_1)F(\tau_1) = \delta^{-2} \overline{F}^T(\tau_1) \overline{F}^T(\tau_1) \le I$, $\delta = \max_{\tau' \in [-T/2, T/2]} \left\| \overline{F}(\tau') \right\|_2 = \max_{\tau' \in [-T/2, T/2]} \left\| \int_0^{\tau'} e^{At} dt \right\|_2 = \left\| \int_0^{T/2} e^{At} dt \right\|_2$

Therefore, we have $\Gamma_1 = D_1 + EFB, \Gamma_2 = D_2 - EFB$.

So the discrete model of the controlled object can be described as the following form:

$$\begin{cases} x_{k+1} = \Phi x_k + (D_1 + EFB)u_k + (D_2 - EFB)u_{k-1} + N_1 G_k \\ y_k = Cx_k \\ z_k = Cx_k + N_1 G_k \end{cases}$$

Where $F^T F \le I$.

Since the state of the controlled object in the network control system is not measurable, a full state observer is designed to reconstruct its state. The steps are as follows:

Step 1: Use the output value $\overline{x}_k$ to correct the value: $\overline{x}_k = \hat{x}_k + L(y_k - C\hat{x}_k)$

Step 2: Calculate the state value $\hat{x}_{k+\tau}$ and control quantity u_k at the time of delay $T+\tau$:

$\hat{x}_{k+\tau} = \Phi_0 \bar{x}_k + \Gamma_0 u_{k-1}$, where $\Phi = e^{AT}$, $\Gamma_1 = \int_0^{T-\tau'} e^{AT} Bdt$, $\Gamma_2 = \int_{T-\tau}^{T} e^{AT} Bdt$, $N = \int_0^T Mdt$.

Step 3: Calculate the state value of the observer at time (K+1)T:

$\hat{x}_{k+1} = \Phi \bar{x}_k + \Gamma_1 u_k + \Gamma_2 u_{k-1} = \Phi \hat{x}_k + (D_1 + EFB)u_k + (D_2 - EFB)u_{k-1} + \Phi LC(x_k - \hat{x}_k)$

Step 4: Define the error variable $e_k = x_k - \hat{x}_k$, $e_{k+1} = \Phi e_k - \Phi LCe_k + N_1 G_k$

Thus, we can obtain the following $z_{k+1} = \Lambda_1 z_k + \bar{N}_1 G_k$.

Where $\Lambda_1 = \begin{bmatrix} \Phi + \Gamma_1 \Omega & \Gamma_1 \Omega + \Gamma_1 \Omega LC & \Gamma_1 \Sigma + \Gamma_2 \\ 0 & \Phi - \Phi LC & 0 \\ \Omega & \Omega LC & \Sigma \end{bmatrix}$, $\Omega = \alpha_k K \Phi_0, \Sigma = \alpha_k K \Gamma_0$, $\bar{N}_1 = \begin{bmatrix} N \\ N \\ 0 \end{bmatrix}$.

2.2. Long network communication delay

When a long-delay event occurs at $m > 1$, the mark is E_2, and the probability is r_2. Assuming that the network-induced delay is greater than one sampling period and the part exceeding one period is equal to the value of the short-delay. Taking $m = 2$ as an example below, the method can be extended to the case of $m > 2$.

The same can be obtained $\Gamma_1 = D_1 + EFB, \Gamma_2 = D_2 - EFB$.

Since the state of the controlled object in the network control system is not measurable, a full state observer is designed to reconstruct its state. The steps are as follows:

Step 1: Use the output value a to correct the value: $\bar{x}_k = \hat{x}_k + L(y_k - C\hat{x}_k)$

Step 2: Calculate the state value $\hat{x}_{k+\tau}$ and the control quantity u_k at the time of delay $T+\tau$:

$\hat{x}_{k+\tau} = \Phi_0 \bar{x}_k + \Gamma_0 u_{k-1}$, where $\Phi_0 = e^{A\tau}$, $\Gamma_0 = \int_0^{\tau} e^{A(kT+\tau-t)} dt$, $u_k = \alpha_k K \hat{x}_{k+\tau}$.

Step 3: Calculate the state value of the observer at time (K+1)T:

$\hat{x}_{k+1} = \Phi \bar{x}_k + \Gamma_1 u_{k-1} + \Gamma_2 u_{k-2} = \Phi \hat{x}_k + (D_1 + EFB)u_{k-1} + (D_2 - EFB)u_{k-2} + \Phi LC(x_k - \hat{x}_k)$

Step 4: Define the error variable $e_k = x_k - \hat{x}_k$, $e_{k+1} = \Phi e_k - \Phi LCe_k + N_2 G_k$

Thus, we can obtain the following $z_{k+1} = \Lambda_2 z_k + \bar{N}_2 G_k$.

Where $\Lambda_2 = \begin{bmatrix} \Phi + \Gamma_1 \Omega & \Gamma_1 \Omega + \Gamma_1 \Omega LC & \Gamma_1 \Sigma + \Gamma_2 \\ 0 & \Phi - \Phi LC & 0 \\ \Omega & \Omega LC & \Sigma \end{bmatrix}$, $\Omega = \alpha_k K \Phi_0, \Sigma = \alpha_k K \Gamma_0$, $\bar{N}_2 = \begin{bmatrix} N \\ N \\ 0 \end{bmatrix}$.

From the above $m = 1$, $m = 2$ derivation, we can get that the method has generality and is suitable for the long time delay of $m > 1$.

3. Stability Analysis

The feedback controller based on the state observer is designed for the above closed-loop system, so that the system meets the following requirements:

(1) When external disturbance $G_k = 0$, the closed-loop system gradually stabilizes.

(2) Under zero initial conditions, the controlled output z_k of the closed-loop system satisfies the following performance index H_∞.

$$\sum_{k=0}^{\infty} E\{\|z_k\|^2\} < \gamma^2 \sum_{k=0}^{\infty} E\{\|G_k\|^2\},$$

Where γ is a given scalar.

Lemma 1: For the asynchronous state system $z_{k+1} = \Lambda_i z_k + \overline{N}_i G_k$, $i = 1,2,\cdots,m$ constrained by the event rate r_i, if there is a Lyapunov function $V(x)$ that satisfies the conditions $\beta_1\|x\|^2 \le V(x) \le \beta_2\|x\|^2$, $(\beta_1, \beta_2 > 0)$ and the scalar $a, a_i, (i = 1,2,\cdots,m)$ satisfies the conditions.

(1) $V(x_{k+1}) - V(x) \le (a_i^{-2} - 1)V(x)$

(2) $a_1^{r_1} \cdots a_m^{m} > a > 0$

Then the solution of the above system satisfies $\lim_{k\to\infty} \alpha^k \|z_k\| = 0$, that is, the system is exponentially stable and the decay rate is $r = \prod_{i=1}^{m} a_i^{r_i}$.

Lemma 2: The following statements are equivalent to the symmetric matrix $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22} \end{bmatrix}$:

(1) $L < 0$

(2) $L_{11} < 0, L_{22} - L_{12}^T L_{11}^{-1} L_{12} < 0$

(3) $L_{22} < 0, L_{11} - L_{12} L_{22}^{-1} L_{12}^T < 0$

Lemma 3: Let W, O, S be a real matrix of appropriate dimensions, where W is a symmetric matrix, then for all matrices F satisfying $F^T F \le I$, the inequality $W + S^T F^T O^T + OFS < 0$ holds, if and only if there is a constant $\varepsilon > 0$ such that $W + \varepsilon O O^T + \varepsilon^{-1} S^T S < 0$.

Remark 1: For the system described above, if there is a scalar $\varepsilon > 0$ and a symmetric positive definite matrix $X \in R^{n\times n}, Y \in R^{n\times n}$, $Z \in R^{n\times n}$, So that the following matrix inequality holds.

$$\begin{bmatrix} -a_i^{-2}X & * & * & * & * & * & * \\ 0 & -a_i^{-2}Y & * & * & * & * & * \\ 0 & 0 & -a_i^{-2}Z & * & * & * & * \\ \Phi + D_1\alpha_k K\Phi_0 & D_1\alpha_k K\Phi_0(LC+I)Y & D_1\alpha_k K\Gamma_2 + D_2 Z & \varepsilon EE^T - X & * & * & * \\ 0 & \Phi Y - \Phi LCY & 0 & 0 & -Y & * & * \\ \alpha_k K\Phi_0 X & \alpha_k K\Phi_0 LCY & \alpha_k K\Gamma_2 Z & 0 & 0 & -Z & * \\ -B\alpha_k K\Phi_0 X & B\alpha_k K\Phi_0(LC+I)Y & \alpha_k K\Gamma_2 Z + Z & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0$$

Where $*$ represents the symmetric item in the symmetric matrix, the system is asymptotically stable.

Proof: By simplifying the formula (3) of Lemma 1, we can get: $V(x_{k+1}) - a_i^{-2}V(x) \le 0$ Select symmetric positive definite matrix $P \in R^{n\times n}, Q \in R^{n\times n}$ $R \in R^{n\times n}$, Construct Lyapunov function:

$$V(Z_k) = x_k^T P x_k + e_k^T Q e_k + u_{k-1}^T R u_{k-1}$$

When the external disturbance $G_k = 0$,

$$V(z_{k+1}) - a_i^{-2}V(z_k) = x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1} + u_{k-1}^T R u_{k-1} - a_i^{-2} x_k^T P x_k - a_i^{-2} e_k^T Q e_k - a_i^{-2} u_{k-2}^T R u_{k-2}$$

$$= z_{k+1}\begin{bmatrix} P & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{bmatrix} z_{k+1} - a_i^{-2} z_k \begin{bmatrix} P & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{bmatrix} z_k$$

Let $\Delta = \begin{bmatrix} P & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{bmatrix}$, then the above formula can be derived:

$$V(z_{k+1}) - a_i^{-2} V(z_k) = z_{k+1} \Delta z_{k+1} - a_i^{-2} z_k \Delta z_k = z_k^T \Lambda^T \Delta \Lambda z_k - a_i^{-2} z_k^T \Delta z_k = z_k^T \left(\Lambda^T \Delta \Lambda - a_i^{-2} \Delta \right) z_k < 0$$

We can get $\Omega = \Lambda^T \Delta \Lambda - a_i^{-2} \Delta < 0$

According to Lemma 2, we can get $\Omega = \begin{bmatrix} -a_i^{-2}\Delta & \Lambda^T \\ \Lambda & -\Delta^{-1} \end{bmatrix} < 0$.

According to Lemma 3: $\Omega < 0$ is equivalent to: there is a constant $\varepsilon < 0$ such that $S + \varepsilon \overline{E}\overline{E}^T + \varepsilon^{-1}\gamma\gamma^T < 0$, where $\overline{E} = \begin{bmatrix} 0 & 0 & 0 & E^T & 0 & 0 \end{bmatrix}$, $\gamma = \begin{bmatrix} (-B\alpha_k K\Phi_0)^T \\ (B\alpha_k K\Phi_0(LC+I))^T \\ (B(\alpha_k K\Gamma_0 + I))^T \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$S = \begin{bmatrix} -a_i^{-2}P & * & * & * & * & * \\ 0 & -a_i^{-2}Q & * & * & * & * \\ 0 & 0 & -a_i^{-2}R & * & * & * \\ \Phi + D_1\alpha_k K\Phi_0 & D_1\alpha_k K\Phi_0(LC+I) & D_1\alpha_k K\Gamma_0 + D_2 & -P^{-1} & * & * \\ 0 & \Phi - \Phi LC & 0 & 0 & -Q^{-1} & * \\ \alpha_k K\Phi_0 & \alpha_k K\Phi_0 LC & \alpha_k K\Gamma_0 & 0 & 0 & -R^{-1} \end{bmatrix}$$

For the above formula, multiply to the left and $diag\{P^{-1}, Q^{-1}, R^{-1}, I, I, I, I\}$ to the right respectively,If $P^{-1} = X, Q^{-1} = Y, R^{-1} = Z$ is combined, the conclusion of Theorem 1 can be obtained.

Remark 2: For $\lambda > 0, 0 \le \beta < 1$, if there is a positive definite symmetric matrix P, Q, R, X, Y, Z and a controller coefficient K matrix, the following inequality holds

$$\begin{bmatrix} -X & * & * & * & * & * & * & * & * & * \\ 0 & Z & * & * & * & * & * & * & * & * \\ 0 & 0 & -Z & * & * & * & * & * & * & * \\ 0 & 0 & 0 & -Y & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ \Phi X & \Gamma_1 H\alpha & \Gamma_2 K\alpha & 0 & N & -X & * & * & * & * \\ 0 & \Gamma_1 H & \Gamma_2 K & 0 & 0 & 0 & -\beta^{-2}X & * & * & * \\ 0 & 0 & 0 & \Phi - \Phi LC & N & 0 & 0 & -Y & * & * \\ 0 & (\alpha+\beta)H - Z & -(\alpha+\beta)H - Z & 0 & 0 & 0 & 0 & 0 & -Z & * \\ CX & 0 & 0 & 0 & N & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0$$

Then the closed-loop system meets the above H_∞ performance index.

Proof: Choose the Lyapunov function in Remark 1 as follows:

$$V_{k+1} - V_k = x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1} + u_{k-1}^T R u_{k-1} - x_k^T P x_k - e_k^T Q e_k - u_{k-2}^T Q u_{k-2}$$
$$= \left[\Phi x_k + \Gamma_1 K\alpha x_{k-1} + (\alpha_{k-1} - \alpha)\Gamma_1 K x_{k-1} + \Gamma_2 K\alpha x_{k-2} + (\alpha_{k-2} - \alpha)\Gamma_2 K x_{k-2} + N_2 G_k\right]^T P$$
$$\left[\Phi x_k + \Gamma_1 K\alpha x_{k-1} + (\alpha_{k-1} - \alpha)\Gamma_1 K x_{k-1} + \Gamma_2 K\alpha x_{k-2} + (\alpha_{k-2} - \alpha)\Gamma_2 K x_{k-2} + N_2 G_k\right] + \left[(\Phi - \Phi LC)e_k + NG_k\right]^T Q$$
$$\left[(\Phi - \Phi LC)e_k + NG_k\right] + \left[\alpha K(x_{k-1} - x_{k-2}) + (\alpha_{k-1} - \alpha)Kx_{k-1} - (\alpha_{k-2} - \alpha)Kx_{k-2}\right]^T R$$
$$\left[\alpha K(x_{k-1} - x_{k-2}) + (\alpha_{k-1} - \alpha)Kx_{k-1} - (\alpha_{k-2} - \alpha)Kx_{k-2}\right] - x_k^T P x_k - e_k^T Q e_k$$

The Bernoulli probability condition satisfies:

$E\{\alpha_k-\alpha\}=0, E\{(\alpha_k-\alpha)^2\}=(1-\alpha)\alpha=\beta^2$

We can get:

$$E\{V_{k+1}|V_k\}-E\{V_k\}+E\{z_k^T z_k\}-\gamma^2 E\{G_k^T G_k\}$$
$$=[\Phi x_k+\Gamma_1 K\alpha x_{k-1}+\Gamma_2 K\alpha x_{k-2}+NG_k]^T P[\Phi x_k+\Gamma_1 K\alpha x_{k-1}+\Gamma_2 K\alpha x_{k-2}+NG_k]+\beta^2[\Gamma_1 Kx_{k-1}+\Gamma_2 Kx_{k-2}]^T P$$
$$[\Gamma_1 Kx_{k-1}+\Gamma_2 Kx_{k-2}]+[(\Phi-\Phi LC)e_k+NG_k]^T Q\times[(\Phi-\Phi LC)e_k+NG_k]+[(\alpha+\beta)Kx_{k-1}-(\alpha+\beta)Kx_{k-2}]^T$$
$$R[(\alpha+\beta)Kx_{k-1}-(\alpha+\beta)Kx_{k-2}]-x_k^T Px_k-e_k^T Qe_k+E\{Cx_k+NG_k\}^T\{Cx_k+NG_k\}-\gamma^2 E\{G_k^T G_k\}$$
$$=\eta_k^T(\Omega+\Theta^T\Pi\Theta)\eta_k=\eta_k^T\Xi\eta_k$$

where $\eta_k=[x_k \quad x_{k-1} \quad x_{k-2} \quad e_k \quad G_k]^T$

$$\Omega=\begin{bmatrix}-P&0&0&0&0\\0&R&0&0&0\\0&0&-R&0&0\\0&0&0&-Q&0\\0&0&0&0&-\gamma^2 I\end{bmatrix},\quad \Theta=\begin{bmatrix}\Phi&\Gamma_1 K\alpha&\Gamma_2 K\alpha&0&N\\0&\Gamma_1 K&\Gamma_2 K&0&0\\0&0&0&\Phi-\Phi LC&N\\0&(\alpha+\beta)K-I&-(\alpha+\beta)K-I&0&0\\C&0&0&0&N\end{bmatrix},$$

$$\Pi=\begin{bmatrix}P&0&0&0&0\\0&\beta^2 P&0&0&0\\0&0&Q&0&0\\0&0&0&R&0\\0&0&0&0&I\end{bmatrix}.$$

By Schur's supplementary lemma, the following matrix can be obtained, which needs to be full Remark 2 can only be obtained by satisfying the condition $\Xi<0$.

$$\begin{bmatrix}-P&*&*&*&*&*&*&*&*&*\\0&R&*&*&*&*&*&*&*&*\\0&0&-R&*&*&*&*&*&*&*\\0&0&0&-Q&*&*&*&*&*&*\\0&0&0&0&-\gamma^2 I&*&*&*&*&*\\\Phi&\Gamma_1 K\alpha&\Gamma_2 K\alpha&0&N&-P^{-1}&*&*&*&*\\0&\Gamma_1 K&\Gamma_2 K&0&0&0&-\beta^{-2}P^{-1}&*&*&*\\0&0&0&\Phi-\Phi LC&N&0&0&-Q^{-1}&*&*\\0&(\alpha+\beta)K-I&-(\alpha+\beta)K-I&0&0&0&0&0&-R^{-1}&*\\C&0&0&0&N&0&0&0&0&-I\end{bmatrix}<0$$

Multiply both sides of the above inequality by $diag\{P^{-1},R^{-1},R^{-1},Q^{-1},I,I,I,I,I,I\}$, and let $X=P^{-1},Y=Q^{-1},Z=R^{-1}$, $H=KR^{-1}$, the controller parameter can be obtained by $K=HZ^{-1}$.

Easy to get when $G_k=0: E\{V_{k+1}|V_k\}=\eta_k^T\Xi\eta_k<-\gamma\eta_k^T\eta_k$

When $G_k\neq 0$ can be obtained: $\sum_{k=0}^{\infty}E\{z_k^T z_k\}<\gamma^2\sum_{k=0}^{\infty}E\{G_k^T G_k\}+E\{V_0\}-E\{V_\infty\}$

Summing k from $0\to\infty$ to the above formula can be obtained:

$$\sum_{k=0}^{\infty}E\{z_k^T z_k\}<\gamma^2\sum_{k=0}^{\infty}E\{G_k^T G_k\}+E\{V_0\}-E\{V_\infty\}$$

According to the zero initial condition $E\{V_0\}=0$, and the closed-loop system is gradually stable, we can get $\sum_{k=0}^{\infty}E\{\|z_k\|^2\}<\gamma^2\sum_{k=0}^{\infty}E\{\|G_k\|^2\}$ certificate is completed.

4. Example Simulation

The matrix of known coefficients is $A=\begin{bmatrix}-1 & -1\\ 6.5 & 0\end{bmatrix}, B=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}, C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, M=\begin{bmatrix}0.6 & 0.4\\ 0.2 & 0.1\end{bmatrix}$.

Set the simulation parameters: the sampling period is $T=0.01s$, the delay is $\tau=0.007s$, the probability of data packet loss is assumed to be $\alpha=0.2$, and the parameter $\lambda=0.34$ is known.

Using the LMI toolbox to solve the linear matrix inequality can be obtained

$$X=\begin{bmatrix}0.0019 & -0.0026\\ -0.0026 & 0.0208\end{bmatrix}, Y=\begin{bmatrix}3.7855 & 0\\ 0 & 3.7855\end{bmatrix}, Z=\begin{bmatrix}0.4129 & 0\\ 0 & 0.4129\end{bmatrix},$$

$$H=\begin{bmatrix}0.1995 & 0\\ 0 & 0.1995\end{bmatrix}, L=\begin{bmatrix}0.4474 & 0.0137\\ 0.0137 & 0.4513\end{bmatrix}$$

Then, we have $Z^{-1}=\begin{bmatrix}2.4217 & 0\\ 0 & 2.4217\end{bmatrix}, K=\begin{bmatrix}0.4831 & 0\\ 0 & 0.4831\end{bmatrix}$

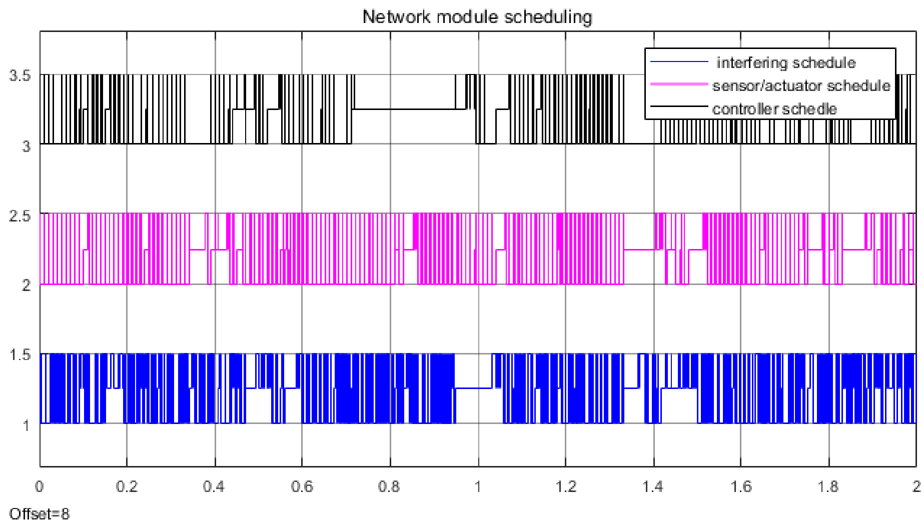

Figure 3: Dispatching situation of each network module.

The network scheduling situation of the system is shown in Figure 3, where the low level of the curve represents the idle state, the high level represents the sending state, and the medium represents the waiting state.

The curve can indicate that the controller node, sensor/actuator node, and interference node all have packet loss and long delay, but the system state can still be stable, which further verifies that the method proposed in this paper is effective.

Given initial conditions $x(0)=\begin{bmatrix}1\\ -1\end{bmatrix}$ and system disturbance $G(t)=0.05\begin{bmatrix}\sin(0.1t)\\ \sin(0.1t)\end{bmatrix}$, the state response curve and control output curve can be obtained through MATLAB/Truetime simulation as shown in Figure 4 and Figure 5, respectively. Through the simulation curve, the system state variables and control output can be gradually stabilized in a small range. Meet the given $H\infty$ condition $\sum_{k=0}^{\infty} E\{\|z_k\|^2\} < \gamma^2 \sum_{k=0}^{\infty} E\{\|G_k\|^2\}$.

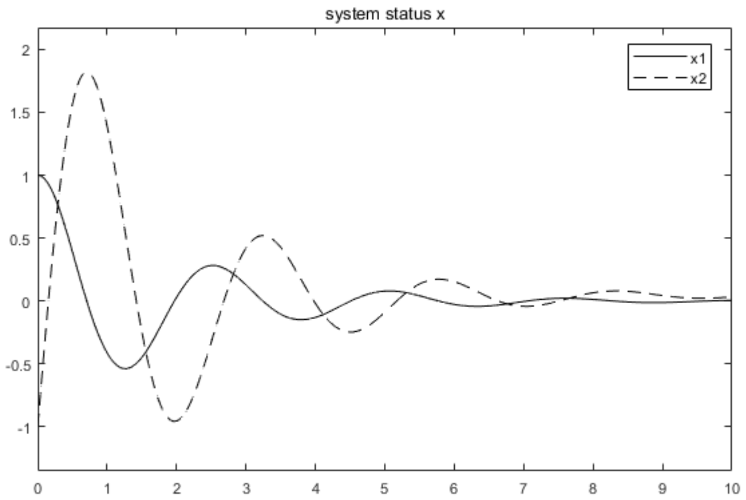

Figure 4: System status response.

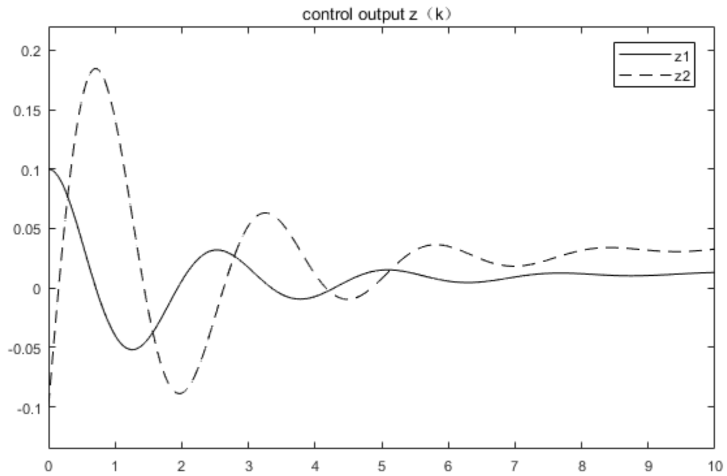

Figure 5: System control output response.

Figure 6 and Figure 7 are the state response curves and control output curves without taking any compensation measures and using the same state feedback controller.

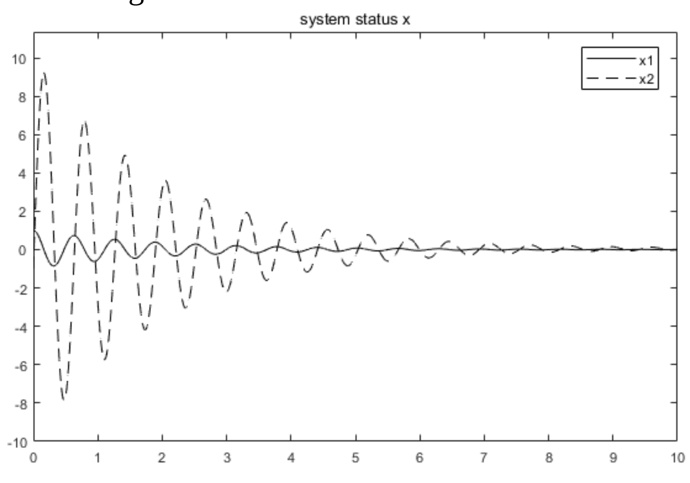

Figure 6: System Status Response without Compensation.

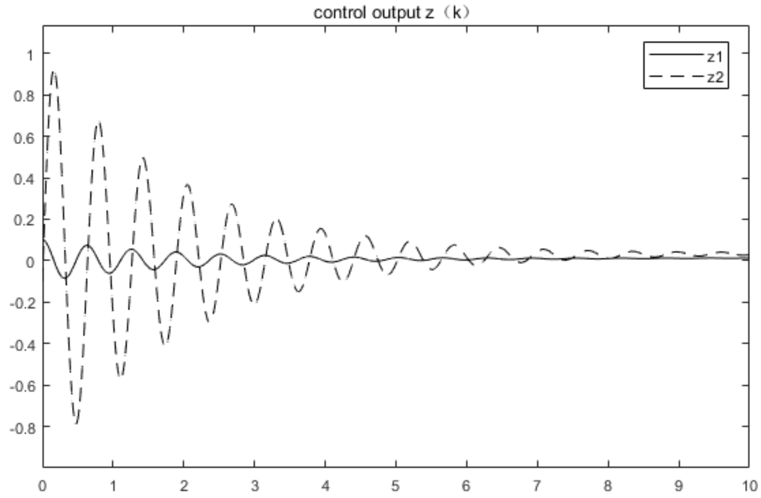

Figure7: System control output response without compensation.

By comparison, adopting the design method of this paper, the curve oscillation amplitude is small, the overshoot is small, and better control performance can be obtained.

5. Conclusions

In order to solve the problems of network communication delay, data packet dropout and interference in the network control system, a new robust model predictive control method with compensation function is proposed. Data packet dropout has the characteristics of Bernoulli sequence random binary distribution; at the same time, the network communication delay in the network control system conforms to the asynchronous dynamic theorem, and the delay less than one sampling period and the delay greater than one sampling period are classified into short-delay and long-delay. Because the state in the system is not measurable, the state observer is used for predictive control, and the $H\infty$ controller strategy is designed based on Lyapunov's theorem and linear matrix inequality. Finally, the effectiveness of the method is verified by simulation. The curve shows that the system can maintain stability in the presence of external interference. Therefore, the $H\infty$ controller designed in this paper is feasible.

References

[1] Ke-You You, Li-Hua Xie. Survey of recent progress in networked control systems. Acta Automatica Sinica,2013, 39(2): 101−118

[2] Rui Wanzhi,Jiang Hanhong,Hou Chongyuan.Survey and prospect of networked control system[J].Information and Control,2012,41(01):83-88.

[3] Zhang Qingling, Zhang Xuefeng.Survey and forecast on networked control systems [J].Information and Control, 2007 (03):364-370.

[4] Ma Weiguo, Shao Cheng. Stochastie Stability for Networked Control Systems [J]. Acta Automatica Sinica, 2007 (08):878-882.

[5] Zhang Y, Ren L T, Xie S S, et al. Robust sliding mode control for uncertain networked control system with two-channel packet dropout [J]. Journal of Central South University, 2019, 26(4):881-892.

[6] Long Chengnian, Dai Shuangfeng, Guan Xinping. Compensation method of networked control system[J].ournal of Jilin University (Information Science Edition),2004(04):438-443

[7] Wang Zhiwen, Pan Wei, Shao Wanghai, Guo Yi. Compensation scheme of a class of networked control systems [J].Journal of Lanzhou University of Technology, 2007(01):84-87.

[8] Liu Yanhong,Yang Dongwei.Design of Mean Square Exponential Stable Controller for Networked Control System Based on Observer[J].Journal of Zhengzhou University (Engineering Science),2018,39(03):10-14.

[9] Zhang Xiaowei,Jiang Shun,Pan Feng.State observer- based non- fragile $H\infty$ control for networked control system. [J].Computer Engineering and Applications, 2017, 53(08):38-43+67.

[10] Ma Yongguang,Chen Wenying,Wang Bingshu,Ge Wei.Compensation of networked control systems with time -delay and data packet losses.Computer Engineering and Applications,2010,46(14):75-78.

[11] Shi Feifei,Huang Ling,Han Jianwei.Delay Compensation for Network Control Systems with Data -dropout.Journal of Harbin University of Science and Technelegy.2017,22(03):36-41.

[12] Liu Yicai,Liu Bin,Zhang Yong,Li Weigang.Stability analysis for nrtworked control systems with bilateral random delay and packet dropout.Control and Decsion,2017,32(09):1565-1573.

[13] Zhu Yijin. $H\infty$ control for NCS with measurement datadropouts and long delay,Nanjing University of Posts and Telecommunications .2016.

[14] Liu A, Zhang W A, Yu L, et al. New results on stabilization of networked control systems with packet disordering [J]. Automatica, 2015, 52(C):255-259.

[15] W. Ren and J. Xiong, "Tracking Control of Nonlinear Networked and Quantized Control Systems With Communication Delays," in IEEE Transactions on Automatic Control, vol. 65, no. 8, pp. 3685-3692, Aug. 2020, doi: 10.1109/TAC.2019.2949102.

[16] Han Jianwei.Research on Time Delay Compensation Strategy of Networked Control System based on True Time,Harbin University of Science and Technology,2018.