

Equity Evaluation of Global Countries based on the Expanded Theil index and Factor Analysis

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Keywords: Theil index, Factor Analysis, equity evaluation

Abstract: Based on the definition of equity, we first refer to the Theil index in economics, which we choose as an evaluation indicator due to its decomposability. The main factors affecting the global disparity among countries are economic indicators, military capability and technology level, etc. Thus, in the first question, we consider quantifying the disparity of countries through factor scores, and then combine it with the Theil index, to reflect the degree of global equity. We selected eight countries as typical samples and conducted factor analysis on them, using SAS to calculate their composite factor scores, and finally classified them according to their factor scores and calculated the Theil index to measure international fairness.

1. Introduction

On the proposition of defining global equity, Wang Xiangying and Hu Yingzhi ^[1] put forward the international equity guidelines based on the study of International Law and the research of the connotation of equity, which provided us with the definition of equity; then, based on the research of different organizational structures in enterprises, Li Chenchen ^[6] put forward a better organizational system structure scheme, which provided us with a reference for building a new organizational structure. So according to what indicators how fair distribution is more especially important, before and after Ge Honglei and Liu Nan ^[2] based on the in-depth discussion of the RAP problem proposed the RAP problem fairness measurement indicators and its selection criteria and Hu Angang et al ^[3]. Based on the historical evaluation of comprehensive national power indicators proposed today's international community evaluation of national power indicators, allowing us to determine the fairness evaluation indicators and original variables.

We will define what global equity is, and propose a global equity model to measure global equity.

2. Model establishment and solution

We introduce the Theil index in economics to define fairness. We first take advantage of the decomposability of the Theil index and use the analogy of comprehensive national power to personal income to expand the Theil index from a measure of income disparity between individuals or regions to a measure of international equity, then combine the results of the comparison of the Gini coefficients between X and Y countries for verification. Finally, based on the evaluation and

comparison of historical data, we derive the general trend of world development.

2.1 Expansion of Theil index

The Theil index is an indicator used to measure inequity and is shown as follows:

$$T = \frac{1}{N} \sum_{i=1}^k \frac{y_i}{y} \ln \frac{y_i}{y} \quad (1)$$

Where y represents personal income, N represents population size, and T represents the Theil index.

The reason why the numerator of the fraction before the summation sign is 1 in the traditional Theil index formula is that the object of judgment is the individual. For a population set of countries, a weighted expression for inequality to judge fairness can be obtained by taking a population size weighting approach.

$$T' = \frac{n_i}{N} \sum_{i=1}^k \frac{y_i}{y} \ln \frac{y_i}{y} \quad (2)$$

Where n represents the population of the corresponding country, k represents the number of countries.

To make the Theil index independent of demographic factors, a logarithmic transformation of the Theil index is conducted to get the corrected Theil index ^[2].

$$T^* = T' / (\ln N - T') \quad (3)$$

2.2 Measurement of comprehensive national power

In a complex world, measuring the comprehensive power of a country is a very arduous and abstract task. We selected several variables for assessment based on the reality that there are many correlated variables, that contain hidden variables that are arduous to interpret, so this paper adopts a factor analysis strategy.

Step 1: Selection of variables

Based on the realistic basis and previous studies [3], we selected the following variables as the original variables. A sample of 8 countries (the U.S., India, Mexico, Dominican Rep., Haiti, Sudan, China, and Chad) was taken as the original sample.

Table 1: Original Variable

Symbol	Original variable	Symbol	Original variable	Symbol	Original variable
c_1	GDP	c_5	Population	c_9	Production per Capita
c_2	Territory Area	c_6	Health	c_{10}	Telephone Lines
c_3	Military	c_7	Education	c_{11}	International Trade
c_4	Technology level	c_8	Official Expenses	c_{12}	Internet Population

Step 2: Calculate the correlation coefficient matrix R and construct up the factorial equation

$$X = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_{12} \end{pmatrix} A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{12.1} & a_{12.2} & \cdots & a_{12.m} \end{pmatrix} F = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{pmatrix} E = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_{12} \end{pmatrix} \quad (4)$$

$$X = AF + E (E \text{ represents the residual matrix})$$

We can find that the sum of the contributions of the first three factors has exceeded 80%, i.e. $\eta = \sum_{i=1}^3 \lambda_i / \sum_{i=1}^{12} \lambda_i > 80\%$. Hence, the number of factors is three.

Step 3: Derive the factor score formula and calculate the composite country power index

$$\begin{cases} f_1 = 0.27c_1 + 0.69c_2 + 0.18c_3 + 0.43c_4 + 0.65c_5 + 0.16c_6 \\ \quad + 0.7c_7 + 0.02c_8 + 0.5c_9 + 0.42c_{10} + 0.69c_{11} + 0.69c_{12} \\ f_2 = 0.65c_1 + 0.67c_2 + 0.03c_3 + 0.14c_4 + 0.6c_5 + 0.52c_6 \\ \quad + 0.33c_7 + 0.97c_8 + 0.67c_9 + 0.69c_{10} + 0.88c_{11} + 0.73c_{12} \\ f_3 = 0.75c_1 + 0.33c_2 + 0.41c_3 + 0.93c_4 + 0.73c_5 + 0.75c_6 \\ \quad + 0.38c_7 + 0.03c_8 + 0.38c_9 + 0.63c_{10} + 0.59c_{11} + 0.97c_{12} \end{cases} \quad (5)$$

Combine the above equation and the composite factor score formula, i.e.

$$F = \sum_{i=1}^3 \lambda_i f_i / \sum_{i=1}^{12} \lambda_i \quad (6)$$

Table 2: Countries' composite power scores in 2017

Country	the U.S.	China	Haiti	Sudan	Mexico	Chad	India	Dominican Rep
Score (F)	0.953	0.86	0.249	0.452	0.693	0.256	0.712	0.483

3. Test of the Equity Measurement Model

According to the citation [2], the Theil index satisfies the scale-invariance, transfer principle, and decomposability principle, having good applicability.

3.1 Historical test

Each era of development has the characteristics of each era of development, so it is very difficult to try to truly achieve fairness with data over time. However, we can compress the modern decades into an interval of relatively constant conditions on the time axis, and use the evaluation indicators we derived earlier to reflect the trend of equity in the modern world.

Based on the resulting factor scores, we can classify countries into four categories: A ($0.75 \leq F < 1$), B ($0.5 \leq F < 0.75$), C ($0.25 \leq F < 0.5$), and D ($0 \leq F < 0.25$). We get a time-dependent Theil index change curve in Figure 1.

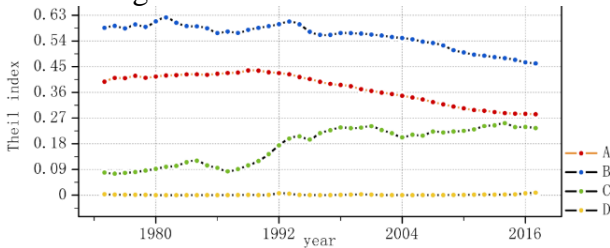


Figure 1: Theil index change curve

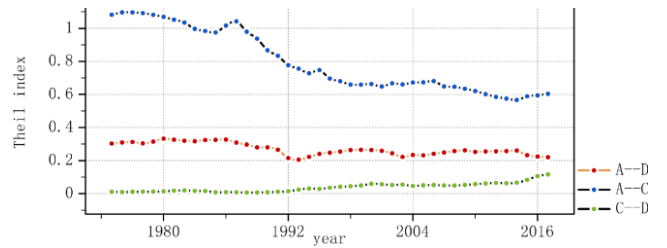


Figure 2: Theil index for different combinations

The smaller the Theil index is, the fairer it is. From Figure 1, we found that in the horizontal comparison, as the years increase, the Theil index of A grade and B grade countries become smaller and fairer; the Theil index of D grade countries is smooth and fairness is stable; while the Theil index of C grade countries becomes larger and more unfair. Then, we found in the longitudinal comparison

that the B-rank curved Theil index is higher than A, while A is higher than C and C is higher than D. Thus, we conclude that B-rank has the highest fairness followed by A, C, and D.

3.2 Regional test

Regional validation inherits the classification of countries in historical validation. Grouping countries of different grades and calculating of Theil index over time in Figure 2. We find that the group of A-grade and D-grade countries produce a large Theil index, indicating very uneven fairness of this combination and in line with the objective laws of reality, thus verifying the objectivity and accuracy of the above results.

3.3 Sensitivity test of selection index

To test the sensitive type of defining global equity using each indicator for each country as defined in this paper, we observe the change in the Thiel index by varying all indicators for C-rank countries up and down by 5% at the same time, using 2017 as an example.

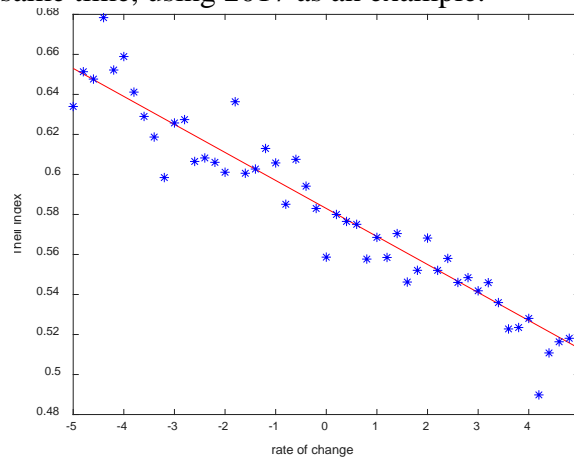


Figure 3: Index change with the rate of change

Figure 3 shows that the Theil index decreases significantly as the rate of change increases, signifying that the Theil index is sensitive to changes in the indicators, which also indicates that fairness is more pronounced with changes in the indicators.

References

- [1] Xiang-Ying Wang, Hu Y. C.. (1990). *The principle of equity in international law and its application*. *Law Review* (04), 39-42. doi: 10.13415/j.cnki.fxpl.1990.04.008.
- [2] Ge, Honglei & Liu, Nan. (2012). *Equity measures in resource allocation and their selection criteria*. *Statistics and Decision Making* (09), 50-53. doi: 10.13546/j.cnki.tjyj.2012.09.014.
- [3] Hu Angang, Gao Yuning, Zheng Yunfeng, Wang Hongchuan. *On the Rise and Fall of Great Powers and Opportunities for China: An Assessment of National Comprehensive National Power [J]*. *Private higher education research*, 2018(2): 98-106.
- [4] Liu, Hui-Hong, Yu, Jie-Ya, Qi, Ming-De, and Mi, Zhong-Chun. (2005). *Cooperative competition game and its solution*. *Forecasting* (02), 73-75+72. DOI:
- [5] Liu, C. Y.. (2006). *The application of "international distributive justice" among countries in the context of globalization*. *International Insights* (06), 36-42. DOI:
- [6] Li, Chen-Chen. (2021). *Research on organizational structure form of engineering general contracting enterprises and its optimization*. *Engineering Economics* (03), 64-67. doi: 10.19298/j.cnki.1672-2442.202103064.