Journal of Sociology and Ethnology (2021)
Clausius Scientific Press, Canada
DOI: 10.23977/jsoce.2021.030822
ISSN 2616-2318 Vol. 3 Num. 8

Discussion on Numerical Algorithm of Initial Value Problem for First Order Ordinary Differential Equation

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Keywords: Differential equation, Discrete, Node

Abstract: In this paper, MATLAB software is used to carry out the calculation research of ordinary differential equation. The calculation ideas of Euler, optimized Euler and classical Kutta are explored respectively, and the calculation process of the three algorithms is analyzed. Through the calculation and analysis of the calculation error of the three algorithms, it is found that: the Euler algorithm has a large error problem; the optimized Euler algorithm forms a large amount of calculation; the calculation process of the three algorithms is analyzed The classical Kutta calculation has high accuracy and can stably complete the calculation program of each group of ordinary differential equations.

1. Introduction

In engineering calculation, research and development activities, we often face the problem of solving differential equations. Only a few differential equations can be solved in elementary equations. In most cases, approximate method is used to solve the problem. Each data element in differential equation has discrete property. When solving the problem, the interval of mathematical element in the equation should be divided into several parts. Based on setting step size, the function solution calculation is carried out. During the solution, the function is placed at several discrete nodes to maintain the equality of node spacing, so as to complete the calculation.

Basic calculation ideas

1. 1 Euler Theory

Euler algorithm is relatively simple. Based on Taylor Theorem, the function calculation is carried out. such as The taylor expression of y(a)at point an can be obtained as follows 1:y(an+1)=y(an)+hy(an)+ $\frac{h\times h}{2!}$ y(ξn), The value range ofξn is(an, an+1). In equation 1, when h is infinite, the calculation error item, namely rn, will be omitted= $\frac{h\times h}{2!}$ y(ξn), Then y(an+1)≈ y(an)+hy(an), then y(an)=f(an, yn), and y(an+1)≈y(an)+hf(an, yn)are derived. Assuming that ym is an approximate value of y(am), and M is 1, 2, . . . , m, then there is(3)yn+1=yn+hf(an, yn).

3:The equation can be calculated in Euler formula, and can obtain(4)yn+1=yn+hf(an, yn), where a is[k, l], y(k)=y0. In Euler algorithm, the calculation of yn+1 is completed with the help of yn. As a single calculation square, the local truncation error is o(h2), and the overall error of the equation is expressed as o(h), which is expressed as the first-order convergence calculation form[1].

1. 2 Optimization of Euler Algorithm

In order to obtain a more accurate calculation method, the integral formula of question type is adopted to replace the integral formula $\int_{an}^{an+1}$ f(z, y(z))dz, Thus, equation 5 is obtained $\int_{an}^{an+1}$ f(z, y(z))dz ≈ $\frac{h}{2}$ {f{an, y(an)+F{an+1, y(an+1)}};from the equation calculation, we can get that:y(an+1)=y(an)+ $\int_{an}^{an+1}$ f(z, y(z))dz, y(an+1)≈y(an)+ $\frac{h}{2}$ {f{an, y(an)+f{an+1, y(an+1)}}. Taking yn as the expression of Y(an)and yn+1 as the expression of y(an+1), we can get the following result:7 yn+1=yn+ $\frac{h}{2}$ {f{an, y(an)+f{an+1, y(an+1)}}.

The calculation method of trapezoidal formula is of second order convergence. Compared with Euler method, the first order calculation program is added. This calculation method has implicit property, so the calculation program is optimized. The optimization method is:obtain the predicted value yn+1 with the help of Euler formula, and the calculation method is yn+1=yn+HF(an, yn);on this basis, the trapezoidal formula is used to complete the numerical correction and obtain the approximate value of the equation, and the calculation method is yn+1=yn+ $\frac{h}{2}$ In the calculation of the approximate value, we can get the estimated value of YN+1, that is, we can get the estimated value of YN+1, that is, when the calculated value of YN+1 is corrected, yn+1=yn+HF(an, yn);when the calculated value of YN+1 is corrected, yn+1=yn+ $\frac{h}{2}$ The Eulerian calculation program{f{an, y(an)+F{an+1, y(an+1)}}}is formed. The Euler calculation program is 9 yn+1=yn+HF(an, yn), yn+1=yn+ $\frac{h}{2}$ In the calculation program, the local error is set as O(H3), thus the optimal Euler algorithm, i. e. the second-order convergence calculation program, is obtained.

1. 3 Classical Kutta Algorithm

In the classical Kutta calculation method, during the actual calculation, the calculation standard for data generation is low. If the accuracy has the same height, the calculation of high-order derivative can be ignored, and the calculation concept is y(an+1)=y(xn)+ $\int_{an}^{an+1}$ f(z, y(z))dz. From the integral equation, we can see that 10: $\int_{an}^{an+1}$ f(z, y(z))dz=f(an+θh, y(an+θh)), and the value range ofθis[0, 1];in the equation, if f(an+θh, y(an+θh)is approximate to the linear combination of functions, then there is yn+1=yn+h $\sum_{m=1}^{s}$ cmf(am, ym)is used to obtain the S-level calculation formula. Combined with the difference requirements of calculation accuracy, the undetermined coefficient can be calculated and a variety of calculation formulas can be derived. Kutta calculation has been widely used in engineering calculation. As a more stable method of calculation program, its local calculation error is set as o(h5). The formula for solving the equation is 11 $y_{an+1}=y_n+\frac{h}{6}(p_1+2p_2+2p_3+p_4)$, $p_1=f(a_n, y_n)$, $p_2=f(a_n+\frac{h}{2}, y_n+\frac{h}{2}p_1)$, $p_3=f(a_n+\frac{h}{2}, y_n+\frac{h}{2}p_2)$, $p_4=f(a_n+h, y_n+hp_3)$.

1. 4 Euler

The calculation method is: [a, b] = ouler (dbfum, axpan, b0, h). In the Euler calculation method, % dbfnu is the function, that is, f (a, b), aspan is used to determine the solution interval [P, q], b0 is the initial value, h is the function step size, a is the function node, and the b value is solved. Then we have: a = aspan (1): h: aspan (2); b (1) = b0; because n = 1: length (a) - 1, then there is a (n + 1) = b (n) + h × feval (dbfun, a (n), b (n)); thus the values of a and b are obtained.

1. 5 Euler Optimization

The calculation method is [a, c] = couluaouler (dcfun, a0, c0, l, i). In the method, % dcfun is used as the representation of function f (a, c), aspan is the solution range [a, b], a0 and c0 are the initial values, l is the step size, a is the node, and c is the solution value. There are a=zeros(1, i+1);c=zeros(1, i+1), a(1)=a(0);c(1)=c(0);Because i=1:i, Then there are a(i+1)=a(i)+l;cbar=c(i)+l×feval(dcfun, a(i), c(i));c(i+1)=c(i)+l/2×(feval(dcfun, a(i), c(i))+feval(dcfun, a(i+1), cbar), The values of a and c are obtained.

1. 6 Classic Kutta

Calculation method:[c, e]=kutta(defun, c0, e0, h, m)). In the method, %defun as a function f(c, e)representation of Cspan is the solution range [k, s], c0 and e0 are the initial values, h is the step size, C is the node, and E is the solution value. Then there are c=zeros(1, m+1);e=zeros(1, m+1), c(1)=c(0);e(1)=e(0);Because m=1:m, There are c(m+1)=c(m)+h;p1=h×fevkl(defun, c(m), e(m));p2=h×fevkl(defun, $c_m+\frac{h}{2}$, $e_m+\frac{h}{2}p_1$);p3=h×fevkl(defun, $c_m+\frac{h}{2}$, $e_m+\frac{h}{2}p_2$);p4=h×fevkl(defun, cm+h, em+hp3);e(m+1)=e(m)+h/6×(p1+2p2+2p3+p4), The values of c and e are obtained.

Case analysis

1. 7 Calculation Example

For example, da /db = y-2b / a, a value range is [0, 1. 4], H value is 0. 2, a (0) value is 1, and the solution equation is a = (2b + 1) - 1.

1. 8 Algorithm Error Analysis

Three calculation methods of Euler, optimized Euler and Kutta method are respectively used to solve the example. The solution of the equation is shown in Fig. 1, and the calculation accuracy of the equation is shown in Fig. 2. In the figure, the plus sign indicates the Euler algorithm, the hollow represents the optimized Euler algorithm, and the solid represents the Kutta algorithm. The coordinate axis corresponding to a parameter is y axis, and that of parameter b is x axis.

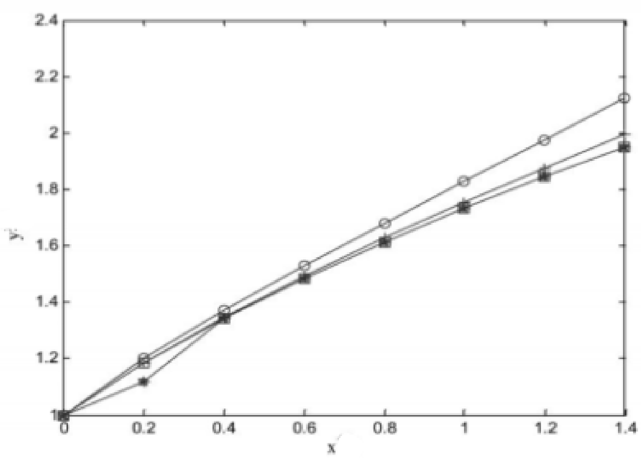

Fig. 1 Equation Solution of Three Calculation Methods

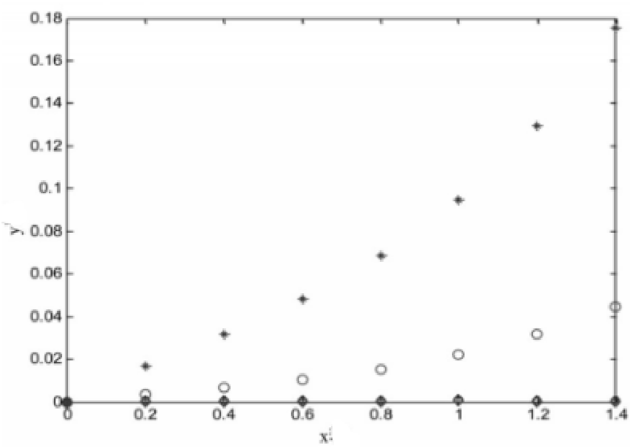

Fig. 2 Calculation Accuracy of Three Calculation Methods

The calculation program is as follows:

dafun=inline(‘a-2b/a’, ‘b’, ‘a’);

[b, a]=ouler(dafun, bspan, b0, h);

[b, a]=youhuaouler(dafun, bspan, b0, h, m);

[b, a]=kutta(dafun, bspan, b0, h, m);

The parameters are brought into three kinds of calculation programs to obtain the solution value of equation in Fig. 1.

1. 8. 1 Solution Analysis

As shown in Figure 1, the optimized Euler calculation method improves the calculation accuracy to a certain extent compared with the Euler solution method, while in the overall performance, the Kutta calculation method performs better [2]. In Figure 1, the Euler algorithm represented by the plus sign has no advantage in solving the curve. The value range of y-axis is [0, 1. 95], and that of x-axis is [0, 1. 4]. In Figure 1, the Kutta algorithm represented by the hollow space performs better in solving the curve.

1. 8. 2 Calculation Accuracy Analysis

As shown in Figure 2, the calculation accuracy relationship of the three algorithms is shown as follows: Kutta > optimized Euler > Euler [3].

As shown in Table 1, the experimental data and calculation accuracy bias of Euler algorithm are shown.

Table 1 Experimental Data And Calculation Accuracy Bias of Euler Algorithm

bn points	Numerical solution of an	a. True value of bn	Accuracy bias
0. 0	1	1	0
0. 4	1. 34212	1. 32251	0. 022
1. 2	1. 87565	1. 84351	0. 031

It can be seen from the data in Table 1 that there is a phenomenon of precision bias in the actual calculation of Euler algorithm, which leads to the problem of calculation error. As shown in Table 2, in order to optimize the experimental data and calculation accuracy bias degree of Euler algorithm.

Table 2 Experimental Data And Calculation Accuracy Bias of Optimized Euler Algorithm

bn points	Numerical solution of an	a. True value of bn	Accuracy bias
0. 0	1	1	0
0. 4	1. 348356	1. 341614	0. 006
1. 2	1. 875487	1. 873707	0. 003

According to the data in Table 2, the optimization algorithm improves the accuracy of calculation to a certain extent, and increases the computational stability of Euler algorithm. As shown in Table 3, the experimental data and calculation accuracy bias of classical Kutta algorithm are shown.

Table 3 Experimental Data And Calculation Accuracy Bias of Classical Kutta

bn points	Numerical solution of an	a. True value of bn	Accuracy bias
0. 0	1	1	0
0. 4	1. 341618	1. 341618	0
1. 2	1. 871849	1. 871849	0

From the data in Table 3 and the curve trend in Fig. 2, it can be seen that the deviation degree between the calculation result and the real value in the process of solving the differential equation is 0, and it performs well in the multi-component point calculation, so as to determine the accuracy and stability of the algorithm.

1. 9 Discussion

Through the actual data and the trial calculation of three algorithms, it is found that:

(1) In the actual calculation of Euler algorithm, there is a phenomenon of precision bias, which leads to the problem of calculation error.

(2) To a certain extent, the optimization algorithm improves the accuracy of calculation and increases the stability of Euler algorithm.

(3) In the process of solving the differential equation, the deviation degree between the calculation results and the real value is 0. In the multi-component point calculation, the classical Kutta performs well, showing the accuracy and stability of the algorithm.

(4) In the classical Kutta and optimal Euler algorithms, the classical Kutta calculation accuracy is higher, and the optimization Euler calculation accuracy is between the Euler and the classical Kutta,

so as to determine the reliability of the classical Kutta calculation method.

2. Conclusion

To sum up, with the help of MATLAB software, the initial calculation program of ordinary differential equation is comprehensively carried out. In the error analysis of calculation results, it is found that: the Euler algorithm has a large error problem; the optimized Euler algorithm forms a large amount of calculation, which improves the calculation accuracy scientifically, which is slightly insufficient compared with the classical Kutta calculation; the classical Kutta calculation has high accuracy, and can stably complete each The calculation program of ordinary differential equations. Compared with the optimized Euler algorithm, the accuracy of the classical Kutta algorithm is increased by 2 times, and the calculation accuracy of the classical Kutta algorithm is increased by 4 times.

3. Acknowledgment

This article is the research result of the online open course project of Yunnan University of Finance and Economics

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