Journal of Artificial Intelligence Practice (2021) Vol. 4: 71-73
Clausius Scientific Press, Canada

DOI: 10.23977/jaip.2020.040211
ISSN 2371-8412

Some Conclusions of the Derivatives of Leibniz triple systems

Gao Rui, Fan Yongyan

Department of Mathmatics, Cangzhou Normal University, Cangzhou, Hebei, 061001, China

Keywords: Leibniz triple system, Centroid, Construction

Abstract: According to the definitions of the centroid and the generalized derivation of Leibniz triple systems, some conclusions of centroid of Leibniz triple systems are developed. In particular, a kind of construction of Leibniz triple system is given in this paper.

1. Introduction

Leibniz triple systems were studied by Bremmner and Sanchez-Ortega, in 2011[1]. In paper [1], by using the algorithm of Kolesnikov and Pozhidaev, the definition of Leibniz triple system was given in a functional form. Cao Yan studied the centroids of Leibniz triple systems in reference [2]. In paper [3], by giving definitions of generalized derivation algebra $GDer(L)$, quasi derivation algebra $QDer(L)$ and quasi centroid algebra $QC(L)$ of a Leibniz triple system L, we have proved the relation and some properties of them. If L is a Leibniz triple system, we can get that $QDer(L)$ is a Lei algebra. We get the quasi centroid algebra $QC(L)$ is a Lie algebra, and also get the sufficient and necessary condition of it. In this paper, some conclusions of centroid $C(L)$ of Leibniz triple systems are developed. In particular, a construction of Leibniz triple system is proved.

2. Definition

Definition 1[1] A vector space L which is defined over a field F is a Leibniz triple system if it satisfies the following two identities:

$\{x,\{y,z,u\},v\} = \{\{x,y,z\},u,v\} - \{\{x,z,y\},u,v\} - \{\{x,u,y\},z,v\} + \{\{x,u,z\},y,v\}$ (1)

$\{x,y,\{z,u,v\}\} = \{\{x,y,z\},u,v\} - \{\{x,y,u\},z,v\} - \{\{x,y,v\},z,u\} + \{\{x,y,v\},u,z\}$

(2) $\forall x,y,z,u,v \in L$, where $\{\cdot,\cdot,\cdot\}$ denotes trilinear operation in L.

Definition 2[2] Let L be a Leibniz triple system. The derivation of L is a linear transformation $D \in End(L)$ of L into L, and satisfies the following

$D(\{a,a',a''\}) = \{Da,a',a''\} + \{a,Da',a''\} + \{a,a',Da''\}, \forall a,a',a'' \in L.$

Definition 3[3] L is a Leibniz triple system. Then f is called generalized derivation of L, satisfying:

$\{fa,a',a''\} = f_1(\{a,a',a''\}) - \{a,f_2a',a''\} - \{a,a',f_3a''\}, \forall f,f_1,f_2,f_3 \in End(L).$

Then all generalized derivation of L are denoted by $GDer(L)$.

Definition 4[3] Suppose that L is a Leibniz triple system and L is defined over the field F. The centroid of L is the transforms on L given by

$$C(L) = \{D \in End(T) \mid D(\{a, a', a''\}) = \{D(a), a', a''\} = \{a, D(a'), a''\} = \{a, a', D(a'')\},$$

for any $a, a', a'' \in L$.

Definition 5[4] Assume the $(L, [,])$ is a binary group, where L is a linear space over the field F. The multiplication $[,]: L \times L \to L$ meets bilinearity. When $\forall x, y, z \in L$ is established, we can get the $[x, [y, z]] = [[x, y], z] - [[x, z], y]$. Thus, $(L, [,])$ is called a Leibniz algebra.

3. Conclusion

Theorem 1 The centroid $C(L)$ is the subalgebra of $GDer(L)$.

Proof According to the Proposition 2.3 in [3], we can get $C(L) \subseteq QDer(L)$. And

from the definition of $QDer(L)$ and $GDer(L)$, we can have the conclusion of $QDer(L) \subseteq GDer(L)$. Then, according to the above conclusion, $C(L) \subseteq GDer(L)$ is established. So, we can get that $C(L)$ is the subalgebra of $GDer(L)$ easily.

Theorem 2 Let L be a Leibniz triple system over a field F. If L has ordinary center, $C(L)$ is the interchange subalgebra of $GDer(L)$.

Proof According to the above conclusion, we can have $\{D_1D_2(a), a', a''\} = \{a, D_1D_2(a'), a''\} = \{a, a', D_1D_2(a'')\} = \{D_2D_1(a), a', a''\}$, for any $D_1, D_2 \in C(L), a, a', a'' \in T$, which implies that $\{(D_1D_2 - D_2D_1)(a), a', a''\} = 0$. Moreover, if the center of T is 0, $[D_1, D_2](x) = 0$, and $[D_1, D_2] = 0$.

Theorem 3 Suppose that $(L, \{,,\})$ is a Leibniz triple system over a field F. For I is unknown, we can define: $\tilde{L} := \{\sum (a \otimes I, b \otimes I^2, c \otimes I^3) : a, b, c \in L\}$. Thus, $\tilde{L}$ with a trilinear operation $\{a \otimes I^i, b \otimes I^j, c \otimes I^k\} = \{a, b, c\} \otimes I^{i+j+k}$,

$\forall a, b, c \in L, i, j, k \in \{1, 2, 3\}$, is a Leibniz triple system.

Proof According to the definition of the Leibniz triple system and the definition of $\tilde{L}$, we can get the following

$$\begin{aligned}
&\{x \otimes I^i, \{y \otimes I^j, z \otimes I^k, \xi \otimes I^m\}, \eta \otimes I^n\} \\
&= \{x, \{y, z, \xi\}, \eta\} \otimes I^{i+j+k+m+n} \\
&= \{\{\{x, y, z\}, \xi, \eta\} - \{\{x, z, y\}, \xi, \eta\} - \{\{x, \xi, y\}, z, \eta\} \\
&\qquad + \{\{x, \xi, z\}, y, \eta\}\} \otimes I^{i+j+k+m+n} \\
&= \{\{x, y, z\}, \xi, \eta\} \otimes I^{i+j+k+m+n} - \{\{x, z, y\}, \xi, \eta\} \otimes I^{i+j+k+m+n} \\
&\qquad - \{\{x, \xi, y\}, z, \eta\} \otimes I^{i+j+k+m+n} + \{\{x, \xi, z\}, y, \eta\} \otimes I^{i+j+k+m+n},
\end{aligned}$$

for any $x, y, z, \xi, \eta \in L; i, j, k, m, n \in \{1, 2, 3\}$. Then, the conclusion of the theorem that $\tilde{L}$ is a Leibniz triple system is proved.

Acknowledgment

Funded by Science and Technology Project of Hebei Education Department(Z2018002).

References

[1] BREMNER M R,SANCHEZ-ORTEGA J. Leibniz triple systems [J]. Communications in Contemporary Mathematics, 2014, 16(1):1350051.

[2] Cao Yan, Du Desheng,Chen Liangyun. The centroid of a Leibniz triple system and its properties [J]. Journal of Natural Science of Heilongjiang University, 2015, 32(5):614-617.

[3] Gao Rui, Zhang Weiwei. Generalized Derivations of Leibniz Triple System[J]. Academic Journal of Humanities& Social Sciences, 2020,11(3):104-111.

[4] Zhou Jia, Wang Zenghui. Generalized derivation of Jordan-Lie algebra [J]. Journal of Jilin University (Natural Science), 2017, 55(4):765-770.

[5] Zhou Jia, Ma Lili. Generalized derivation of Leibniz algebra [J]. Journal of Jilin University (Natural Science), 2016, 54(6):1221-1225.

[6] MA Y, CHEN L. Some Structures of Leibniz triple systems [J]. arXiv preprint arXiv: 1407.3978, 2014.

[7] Zhao Guanhua. Generalized derivation of Lie triple system [J]. Journal of Liaocheng University (Natural Science), 2004, 17(3):19-20.