

Research on the Rock-paper-scissors Game and Cooperation

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Abstract: The paper requires that an extended model of May-Leonard with more than four participants be established based on the knowledge of the May-Leonard model, and there are some differences between the two models. For the May-Leonard model, a three-participant model with only three participants is established on the basis of Logistic-Volterra, and MATLAB software is used to solve the model for the change of the number of three participants over time. For the four-participant model with four participants, based on the established three-participant model, introduce the fourth participant to establish a four-participant model, and use MATLAB software to run Gaussian Fitting on the model to obtain four number of participants The relationship between changes over time, and thus the changes in the number of four participants over time can be derived.

1. Introduction

People have more or less participated in the game of rock-paper-scissors. Simply put, three factors restrict each other to achieve a stable balance. From a macro perspective, an environment is full of various populations and differences. The objective factors of these populations restrict each other instead of simply dictating one subordinate relationship. At the same time, they overlap with objective factors to form a relatively stable system.

2. Model establishment and solution

2.1 Analysis of the problem

This question requires the establishment of a model similar to the May Leonard model with more than three participants, so the model should be established and solved based on the May Leonard model. Therefore, the key to the problem is to model and solve the simple may Leonard model, that is, a dominates B, B dominates C, and C dominates A. First, select two of the three participants arbitrarily, which conforms to the Logistic-Volterra, and directly establish the Logistic-Volterra for both. It can be concluded that when there is no third participant the relationship between the growth rate of the participants changes with time, and the third participant is inserted at this time, and the relationship between the number of the three participants with time is obtained based on the parameters of the influence between the two.

For the expanded model with more than three participants, this article first selects only four participants to analyze and solve. Analogous to the established May-Leonard model, this article first discusses the relationship between two participants and then introduces the method of the third participant. Based on the May-Leonard model, this paper introduces the fourth participant to obtain the relationship of the number of four participants over time according to the parameters of the influence between the two.

2.2 Model I: Three-participant model

There are three participants in the system. They satisfy the game relationship of A dominating B, B dominating C, and C dominating A.

When B and C are observed separately, the predator-predator model is satisfied between B and C. Use the following method to establish the model. Assuming that C does not exist, then the number of B will decrease due to lack of food, then

$$\frac{dy}{dx} = -ay. \quad (1)$$

But in fact, in this system, BC has an interactive relationship. The number of C changes with the number of B, and the number of B changes with the number of C. This kind of influence exists from the beginning to the end. It can be expressed by the product of the two quantities, so a more accurate model should be written

$$\frac{dy}{dx} = -ay + byz \quad (2)$$

On the contrary, if there is no B, then the number of C will explode because there is enough space and no natural enemies, then:

$$\frac{dz}{dt} = dz. \quad (3)$$

It can be seen from the above that the influence of B on C has always existed, so this interaction relationship is still expressed by the product which is proportional to the number of the two. Write a more accurate model

$$\frac{dz}{dt} = dz - cyz. \quad (4)$$

At this time, the third participant a is introduced, and a also has an impact on BC. At this time, this interaction relationship is expressed by the product of the number of the two, and the final model is written as follows:

$$\frac{dy}{dt} = -ay + byz - gxy. \quad (5)$$

$$\frac{dz}{dt} = dz - cyz + hxz. \quad (6)$$

Based on the assumption that the sum of all participants' capital can be written down

$$x = c_1 - y - z. \quad (7)$$

Assign values to the parameters involved in the model assumption process within a reasonable range, as follows:

$$a = 1, b = 0.8, c = 0.7, d = 1.5, g = 0.6, h = 0.7, c_1 = 12. \quad (8)$$

Use MATLAB to solve the implicit function

$$y = -0.013t^6 + 0.19t^5 - 1.1t^4 + 3.3t^3 - 5.2t^2 + 4.1t + 3.9. \quad (9)$$

$$z = 12 - 1.005e^{-11.19t} + 0.013t^6 - 0.19t^5 + 1.1t^4 - 3.3t^3 + 5.2t^2 - 4.1t + 0.876. \quad (10)$$

It can be seen from the image of the number of B changes over time that the number of B has just begun to show explosive growth and finally stabilized at 5.5.

It can be seen from the image that the number of C has increased in the first few short periods of time, and after reaching the peak, it begins to decrease and finally stabilizes at 4.3.

By taking the value of t , the value of y and z at that time can be obtained, and then the value of x at that time can be obtained according to the sum of capital of all participants. Therefore, the value of x can be obtained by assigning t in $[0,5]$ Perform curve fitting on the obtained x value to obtain the function of x with respect to t

$$x = 1.005e^{-11.19t} + 3.024. \quad (11)$$

It can be seen from the image that the number of A has been declining from the beginning, and finally stabilized at 3.2.

In order to visually observe the random time variation of the pros and cons of the three participants, as well as the number trend of the three participants, the number of the three participants over time is drawn in the same image.

It can be seen that when $t < 0.25$, B and C are in advantage, and the number keeps increasing, while the number of disadvantages of A keeps decreasing. When $t > 0.25$, the number of A tends to be stable and the number remains basically unchanged, and the number of C changes from advantage to disadvantage. It starts to decline, and the number of B continues to maintain its advantage. When $t = 1$, the numbers of A, B, and C all stabilize and the system reaches equilibrium.

2.3 Model II : Four-participant model

In the three-participant model, the fourth model D is introduced, D is dominated by C, and D can dominate A and B.

In the model, D has a direct and continuous impact on ABC. At this time, the product of the two numbers is proportional to the product to express this interactive relationship. The model written is:

$$x = c_1 - y - z - rxw. \quad (12)$$

$$\frac{dz}{dt} = dz - cyz + hxz + ewz. \quad (13)$$

$$w = c_2 - x - y - z. \quad (14)$$

But D can also have an indirect influence on A through B and C, so at this time, the quantitative product proportional to the three is used to express this influence, so as to adjust the model of A:

$$\frac{dy}{dx} = -ay + byz - gxy - fyw + ceyzw + gixwy. \quad (15)$$

Assign values to the parameters involved in the model assumption process within a reasonable range, as follows:

$$f = 0.5, e = 0.6, i = 0.7, c_2 = 16. \quad (16)$$

Bring the parameters into the model to get the following differential equation:

$$\frac{dy}{dt} = -y + 0.8yz - 0.6xy - 0.5yw + 0.42 = 0.6yzyzw + 0.42xwy. \quad (17)$$

$$\frac{dz}{dt} = 1.5z - 0.7yz + 0.7xz + 0.6yz. \quad (18)$$

Substituting the obtained relationship into the derived implicit function equation of z (w , x , y , t), the relationship between the number of z and time t can be obtained, and the relationship is as follows

$$\frac{dz}{dt} = 1.5z - 0.7yz + 0.7xz + 0.6yz. \quad (19)$$

Gaussian fitting is performed on $x(t)$, $y(t)$, $w(t)$ by the above method to obtain the relationship between the number of A , B , C , and D over time:

$$z = 3.856e^{-1.501t} + 0.06974. \quad (20)$$

$$x = 6.586 \times 10^4 e^{-\left(\frac{t-62.64}{10.41}\right)^2} + 5.984e^{-\left(\frac{t-1.746}{1.421}\right)^2} + 1.864e^{-\left(\frac{t-0.7546}{0.3964}\right)^2} + 3.572e^{-\left(\frac{t-0.1773}{0.3886}\right)^2}. \quad (21)$$

$$w = -9.804 \times 10^{13} e^{-\left(\frac{t+3.736}{0.6587}\right)^2} + 1.236e^{-\left(\frac{t-0.7504}{0.741}\right)^2} - 2.425e^{-\left(\frac{t-0.7055}{1.079}\right)^2} - 1.167e^{-\left(\frac{t-2.968}{1.067}\right)^2}. \quad (22)$$

$$y = 10.98e^{-\left(\frac{t-2.907}{1.439}\right)^2} + 6.967e^{-\left(\frac{t-1.204}{0.6497}\right)^2} + 1.091 \times 10^{15} e^{-\left(\frac{t+23.33}{4.088}\right)^2} + 0.9568e^{-\left(\frac{t-1.981}{0.2944}\right)^2} + 4.687e^{-\left(\frac{t-0.4871}{0.4508}\right)^2}. \quad (23)$$

Make an image of the changes in the number of four participants A , B , C , D over time as shown below:

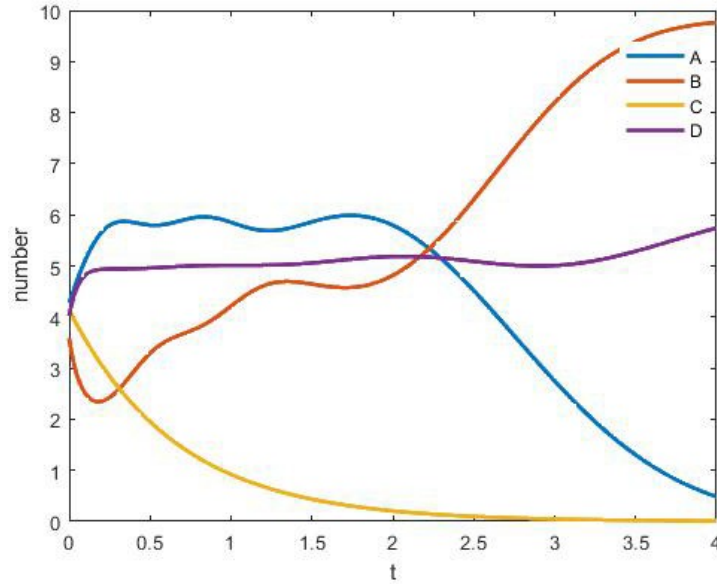


Figure 1: The number of ABCD four participants changes over time.

It can be seen from the image.

When $t < 0.15$, the number of advantages A and D are increasing, and the number of disadvantages B and C is decreasing.

When $0.15 < t < 0.45$, A is still in the advantage and the number of C is still in the inferior is continuously increasing, but B has completed the counterattack and turned from inferiority to advantage, the number began to increase, and D has been stable afterwards.

When $0.45 < t < 2$, A and D tend to be stable, and the number fluctuates in a small range, C is still at a disadvantage, and the number continues to decrease until extinction, and B is still at the advantage and the number continues to increase.

When $t > 2$, the number of A 's turning into inferiority continues to decrease until extinction, while the number of B 's advantages continues to increase and finally stabilizes.

After the above analysis, it can be seen that the four-participant model under this parameter cannot become stable, and will eventually lead to two participants out of the game. Therefore, the currently selected parameters are not the most promising solution. To optimize the sensitivity analysis.

Observing and analyzing the three-participant model and the four-participant model, it can be seen that when there are only three participants, under given parameters or when the parameters fluctuate in a small range, the system will eventually become stable, and the three will restrict each other to achieve balance. However, the four player model is slightly different. Under the given parameters, the two players will eventually be out of the game. Therefore, it can be seen that the given parameters, that is, the interaction between the participants, plays a decisive role in the final balance of the system.

3. Model Evaluation

3.1 Advantages of the model

1. GA has global optimization and can quickly solve all solutions in space.
2. GA is malleable, easy to combine with other algorithms, has good convergence and high robustness.
3. Gaussian Fitting is very simple and quick to calculate integral.

3.2 Disadvantages of the model

The GA has limited search capabilities for new spaces and is easy to converge to a local optimal solution.

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