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Rough Direct Product of Fuzzy Semi-Groups

Hongjie Li

School of Mathematics and Statistics, Zhoukou Normal University, 466001, China

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Abstract: using fuzzy points as a tool, this paper applies the concept of rough set to the theory of fuzzy semi-groups, gives the definition of rough direct product through a congruence relation on fuzzy semi-groups, discusses some properties of rough direct product under fuzzy congruence relation, and further supplements and perfects the rough set theory in fuzzy semi-groups

1. Introduction

As we all know, the concept of fuzzy point proposed by Chinese scholar Liu Yingming plays an important role in the study of fuzzy topology. In 1981, Qi zhenkai introduced this concept into the research of algebra and proposed fuzzy subgroups based on fuzzy points. Since then, he has made a lot of research on fuzzy semi-groups, fuzzy groups and fuzzy subgroups by using fuzzy points, and obtained good conclusions. In this paper, the concept of rough set is applied to the theory of fuzzy semi-groups, the rough direct product under fuzzy congruence relation is discussed, and its fuzzy algebraic structure is characterized.

2. Basic Concepts

Definition 1.1[1] let S be a semi-group, $\mu \in F(S)$, if $\forall x_\alpha, y_\beta \in \mu$ have $x_\alpha y_\beta = (xy)_{\min\{\alpha,\beta\}} \in \mu$, then μ is a fuzzy subsemi-group of S.

Definition 1.2[2] let μ be a fuzzy semi-group on S and ρ be a fuzzy equivalence relation on S if ρ satisfies the following conditions: for any fuzzy point $x_\alpha, y_\alpha, z_\beta \in \mu$, there are

$$(x_\alpha, y_\alpha) \in \rho \Rightarrow \left(x_\alpha z_\beta, y_\alpha z_\beta\right) \in \rho, \left(z_\beta x_\alpha, z_\beta y_\alpha\right) \in \rho,$$

Then ρ is called a Fuzzy congruent relation over μ.

Definition 1.3[2] Let ρ be the Fuzzy congruent relation on Fuzzy semi-group μ, $\forall x_\alpha \in \mu$,, let the congruent class of x_λ be

$$\left[x_\alpha\right]_p = \left\{y_\alpha \in S \left| \left(x_\alpha, y_\alpha\right) \in \rho\right.\right\}$$

Definition 1.4[3] Let A be any Fuzzy subset of S, then the ρ approximation and ρ approximation of A are expressed as:

$$\rho_-(A) = \left\{x_\lambda \in S \left| \left[x_\lambda\right]_p \subseteq A\right.\right\} = \cup\left\{\left[x_\lambda\right]_p \left| \left[x_\lambda\right]_p \subseteq A\right.\right\},$$

$\rho^{-}(A)=\left\{x_{\lambda}\in S\left|[x_{\lambda}]_{p}\cap A\neq\phi\right.\right\}=\cup\left\{[x_{\lambda}]_{p}\left|[x_{\lambda}]_{p}\cap A\neq\phi\right.\right\}$, $\rho(A)=\left(\rho_{-}(A),\rho^{-}(A)\right)$ is called Fuzzy rough set.

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Hong-jie Li (1979-), female, born in Zhoukou, Henan province, associate professor, School of Mathematics and Statistics, Zhoukou Normal University. Her research interest is fuzzy set and rough set theory

Definition1.5[4] Let S be a semi-group, and the multiplication of Fuzzy points defined on S is as follows:

$\forall x_{\lambda},y_{u}\in S,x_{\lambda}y_{u}=(xy)_{\lambda\wedge u}$

Definition1.6[4] Let A,B be two nonempty Fuzzy subsets on Fuzzy semi-group S , and let the Fuzzy direct product on S be $AB=\left\{x_{\lambda}y_{u}\left|x_{\lambda}\in A,y_{u}\in B\right.\right\}$

Definition1.7[3] A Fuzzy congruent relation ρ on a Fuzzy semi-group S is said to be complete if the relation to $\forall x_{\lambda},y_{\lambda}\in S,[x_{\lambda}]_{\rho}[y_{\lambda}]_{\rho}=[(xy)_{\lambda}]_{\rho}$

Definition 1.8[5] Let ρ be a Fuzzy congruent relation on semi-group S , $\rho_{-}(A),\rho^{-}(A),\rho_{-}(B),\rho^{-}(B)$ be a nonempty coarse subset of S , $A,B\subseteq S$, then the coarse direct product on S is denoted as:

$\rho_{-}(A)\rho_{-}(B)=\left\{x_{\lambda}y_{u}\in S\left|x_{\lambda}\in\rho_{-}(A),y_{u}\in\rho_{-}(B)\right.\right\}$

$\rho^{-}(A)\rho^{-}(B)=\left\{x_{\lambda}y_{u}\in S\left|x_{\lambda}\in\rho^{-}(A),y_{u}\in\rho^{-}(B)\right.\right\}$

Definition 1.9[5] Let ρ_{1},ρ_{2} be a Fuzzy congruent relation on semi-group S , $\rho_{1-}(A),\rho^{1-}(A),\rho_{2-}(A),\rho^{2-}(A)$ be a nonempty coarse subset of S , $A\subseteq S$, then the coarse direct product on S is denoted as:

$\rho_{1-}(A)\rho_{2-}(A)=\left\{x_{\lambda}y_{u}\in S\left|x_{\lambda}\in\rho_{1-}(A),y_{u}\in\rho_{2-}(A)\right.\right\}$

$\rho^{1-}(A)\rho^{2-}(A)=\left\{x_{\lambda}y_{u}\in S\left|x_{\lambda}\in\rho^{1-}(A),y_{u}\in\rho^{2-}(A)\right.\right\}$

Lemma1.1 Let ρ_{1},ρ_{2} be a Fuzzy congruent relation on semi-group S , $\rho_{1}(A),\rho_{2}(A)$ be a nonempty coarse subset of S , then $\rho_{1}\subseteq\rho_{2}\Rightarrow\rho_{1-}(A)\subseteq\rho_{2-}(A),\rho_{1}^{-}(A)\subseteq\rho_{2}^{-}(A)$.

Lemma1.2 Let ρ_{1},ρ_{2} be the complete congruences on Fuzzy semi-group S , then $\rho_{1}\cap\rho_{2}$ is the complete congruences on Fuzzy semi-group S .

Lemma 1.3 Let ρ_{1},ρ_{2} be the congruent relation on Fuzzy semi-group S and $\rho_{1}\rho_{2}$ be the compound of the relation, then $\rho_{1}\rho_{2}$ is the congruent relation on Fuzzy semi-group S if and only if $\rho_{1}\rho_{2}=\rho_{2}\rho_{1}$.

3. Main Results

Theorem2.1 Let ρ be a complete congruent relation on Fuzzy semi-group S and A,B be a nonempty Fuzzy subset on S , then $\rho_{-}(A)\rho_{-}(B)\subseteq\rho_{-}(AB)$..

Prove that let $z_{\gamma}\in\rho_{-}(A)\rho_{-}(B)$, exist $x_{\lambda}\in\rho_{-}(A),y_{u}\in\rho_{-}(B)$, such that $z_{\gamma}=x_{\lambda}y_{u}$, so

$\left[x_\lambda\right]_\rho \subseteq A, \left[y_\mu\right]_\rho \subseteq B$, and ρ are complete congruences on Fuzzy semi-group S , so $\left[x_\lambda\right]_\rho \left[y_\mu\right]_\rho = \left[x_\lambda y_\mu\right]_\rho \subseteq AB$, that is, $\left[z_\gamma\right]_\rho \subseteq AB$,so $z_\gamma \in \rho_-(AB)$.

Theorem 2.2 Let ρ be a complete congruent relation on Fuzzy semi-group S and A, B be a nonempty Fuzzy subset on S , then $\rho^-(A)\rho^-(B) \subseteq \rho^-(AB)$.

Prove that if $z_\gamma \in \rho^-(A)\rho^-(B)$, then there exists $x_\lambda \in \rho^-(A), y_\mu \in \rho^-(B)$, such that $z_\gamma = x_\lambda y_u$, therefore there exists $a_s \in \left[x_\lambda\right]_\rho \cap A, b_t \in \left[y_\mu\right]_\rho \cap B$, , and ρ is a complete congruent relation on Fuzzy semi-group S , so $a_s b_t \in \left[x_\lambda\right]_\rho \left[y_\mu\right]_\rho = \left[x_\lambda y_\mu\right]_\rho = \left[z_\gamma\right]_\rho$,that is, $a_s b_t \in \left[z_\gamma\right]_\rho \cap AB$, and so $z_\gamma \in \rho^-(AB)$.

Theorem 2.3 Let ρ_1, ρ_2 be a complete congruent relation on Fuzzy semi-group S and A be a nonempty Fuzzy subset on S, then $\left(\rho_1 \cap \rho_2\right)^-(A) \subseteq \rho_1^-(A) \cap \rho_2^-(A)$ ..

It is proved that from Lemma 1.2, $\rho_1 \cap \rho_2$ is a complete congruent relation on Fuzzy semi-group S. If $z_\gamma \in \left(\rho_1 \cap \rho_2\right)^-(A)$, then $\left[z_\gamma\right]_{\rho_1 \cap \rho_2} \cap A \neq \varnothing$, so exists $x_\lambda \in \left[z_\gamma\right]_{\rho_1 \cap \rho_2} \cap A$, thus $\left(x_\lambda, z_\mu\right) \in \rho_1 \cap \rho_2$, $\left(x_\lambda, z_\mu\right) \in \rho_1$ and $\left(x_\lambda, z_\mu\right) \in \rho_2$; And because $x_\lambda \in A$, then $x_\lambda \in \left[z_\gamma\right]_{\rho_1} \cap A$, and $x_\lambda \in \left[z_\gamma\right]_{\rho_2} \cap A$, therefore $z_\gamma \in \rho_1^-(A) \cap \rho_2^-(A)$, that $\left(\rho_1 \cap \rho_2\right)^-(A) \subseteq \rho_1^-(A) \cap \rho_2^-(A)$.

Theorem 2.4 Let ρ_1, ρ_2 be a complete congruent relation on Fuzzy semi-group S and A be a nonempty Fuzzy subset on S, then $\left(\rho_1 \cap \rho_2\right)_-(A) = \rho_{1-}(A) \cap \rho_{2-}(A)$.

Prove the first: : $\rho_{1-}(A) \cap \rho_{2-}(A) \subseteq \left(\rho_1 \cap \rho_2\right)_-(A)$.

From Lemma 1.2 we know that $\rho_1 \cap \rho_2$ is a complete congruent relation on Fuzzy semi-group S. Because $\rho_1 \cap \rho_2 \subseteq \rho_1$ knows $\rho_{1-}(A) \subseteq \left(\rho_1 \cap \rho_2\right)_-(A)$, similarly $\rho_{2-}(A) \subseteq \left(\rho_1 \cap \rho_2\right)_-(A)$, so $\rho_{1-}(A) \cap \rho_{2-}(A) \subseteq \left(\rho_1 \cap \rho_2\right)_-(A)$.

Prove again : $\left(\rho_1 \cap \rho_2\right)_-(A) \subseteq \rho_{1-}(A) \cap \rho_{2-}(A)$.

Let $z_\gamma \in \left(\rho_1 \cap \rho_2\right)_-(A)$, that is, $\left[z_\gamma\right]_{\rho_1 \cap \rho_2} \subseteq A$, so $\left[z_\gamma\right]_{\rho_1} \subseteq A$ and $\left[z_\gamma\right]_{\rho_2} \subseteq A$, that is, $z_\gamma \in \rho_{1-}(A)$ and $z_\gamma \in \rho_{2-}(A)$, so $z_\gamma \in \rho_{1-}(A) \cap \rho_{2-}(A)$. To sum up $\left(\rho_1 \cap \rho_2\right)_-(A) = \rho_{1-}(A) \cap \rho_{2-}(A)$.

Theorem 2.5 Let ρ_1, ρ_2 be congruent relation on Fuzzy semi-group S , $\rho_1\rho_2 = \rho_2\rho_1$, if A is a Fuzzy subsemi-group of S , then .

Prove $z_\gamma \in \rho_{1-}(A)\rho_{2-}(A)$, there exists $x_\lambda \in \rho_{1-}(A), y_\mu \in \rho_{2-}(A)$, so that $z_\gamma = x_\lambda y_u$, there exists $a_s \in \left[x_\lambda\right]_{\rho_1} \subseteq A, b_t \in \left[y_\mu\right]_{\rho_2} \subseteq A$, , A is a Fuzzy subsemi-group of S, $a_s b_t \in A$, is known, and ρ_1, ρ_2 is congruent relation on Fuzzy semi-group S, and $\left(x_\lambda, a_s\right) \in \rho_1, \left(y_\mu, b_t\right) \in \rho_2$, so $\left(x_\lambda y_\mu, a_s y_\mu\right) \in \rho_1, \left(a_s y_\mu, a_s b_t\right) \in \rho_2$, that is $a_s b_t \in \left[x_\lambda y_\mu\right]_{\rho_1\rho_2}$, so there is $a_s b_t \in \left[x_\lambda y_\mu\right]_{\rho_1\rho_2} \subseteq A$, so $z_\gamma = x_\lambda y_\mu \in \left(\rho_1\rho_2\right)_-(A)$.

Theorem 2.6 Let ρ_1, ρ_2 be the congruent relation on Fuzzy semi-group S, $\rho_1\rho_2 = \rho_2\rho_1$, if A is

A Fuzzy subsemi-group of S, then $\rho_1^-(A)\rho_2^-(A) \subseteq (\rho_1\rho_2)^-(A)$.

Prove $z_\gamma \in \rho_1^-(A)\rho_2^-(A)$, then there is $x_\lambda \in \rho_1^-(A), y_\mu \in \rho_2^-(A)$, so that $z_\gamma = x_\lambda y_u$, so there is $a_s \in [x_\lambda]_{\rho_1} \cap A, b_t \in [y_\mu]_{\rho_2} \cap A$,. It can be seen from the fuzzy subsemi-group with A as S that $a_s b_t \in A$, Because ρ_1, ρ_2 is the congruence relationship on Fuzzy semi-group S, and $(x_\lambda, a_s) \in \rho_1, (y_\mu, b_t) \in \rho_2$, so $(x_\lambda y_\mu, a_s y_\mu) \in \rho_1, (a_s y_\mu, a_s b_t) \in \rho_2$, that is, $a_s b_t \in [x_\lambda y_\mu]_{\rho_1\rho_2}$, so there are $a_s b_t \in [x_\lambda y_\mu]_{\rho_1\rho_2} \cap A$, so $z_\gamma = x_\lambda y_\mu \in (\rho_1\rho_2)^-(A)$, thus $\rho_1^-(A)\rho_2^-(A) \subseteq (\rho_1\rho_2)^-(A)$..

4. Conclusion

By giving the above theorems and proofs, this paper discusses the application of rough set theory in fuzzy semi-groups, puts forward the rough direct product in fuzzy semi-groups, perfects the theory of rough direct product, qualitatively describes some of its algebraic structures, and enriches the theory of rough direct product in semi-groups.

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