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Optimization of Economic Management Dynamic System Based on Differential Equation

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Abstract: In actual social life, people often associate ordinary differential equations with social economy. Under the research of many researchers, great achievements and progress have been made in the use of ordinary differential equations. Among them, the maximum analysis chart of economic target product often appears in relevant publications of “economic management”, “technical economy”, “system engineering”, “operational research” and other disciplines, and is widely used in qualitative and quantitative analysis of economic management dynamic system optimization. This paper attempts to analyze the maximum analysis chart of economic target product, so as to make the maximum analysis chart of economic target product more scientific, practical and effective in the optimization analysis of economic management dynamic system. Complex and Abstractproblems in economics are expressed in mathematical language, and practical problems are closely combined with mathematics.

1. Introduction

Due to the development of modern production, quantitative analysis in economics has made great progress. Some branches of mathematics, such as mathematical analysis, linear algebra, probability statistics, differential equations, etc., have entered economics, and new branches such as mathematical statistics, econometrics, and economic cybernetics have emerged [1-2]. These new branches are usually called quantitative economics. Differential equations have a wide range of research and a long history. Newton and Leibniz pointed out their mutual inverse when they created differential and integral operations. In fact, this solved the simplest problem of solving the differential equation y=f(x) [3]. When people use differentiation to solve problems in economics, they find that their qualitative analysis and quantitative analysis of economic problems are rigorous and credible, so a large number of differential equations emerge.

This paper attempts to analyze the maximum analysis chart of economic target product, so as to make the maximum analysis chart of economic target product more scientific, practical and effective in the optimization analysis of economic management dynamic system.

2. Application of Unitary Calculus in Economic Management

As we know, commodity prices mainly depend on three factors: commodity value, market competition and demand for products. In commodity pricing, demand factors should be considered. Generally speaking, price level has an impact on demand rise and fall, while low price leads to demand rise and high price leads to demand decline. The ratio of demand rise and fall rate to price change rate is called price demand elasticity coefficient, which is expressed by E_P. E_P value calculation formula is as follows:

$$E_P = \frac{\Delta Q / Q}{\Delta P / P} \quad (1)$$

In the formula, Q is the demand and ΔQ is the change of demand; P is the original price; ΔP is the amount of price change; $\Delta Q / Q$ is the rate of demand rise and fall; $\Delta P / P$ is the rate of price change.

Economic lot size method is a method to determine reasonable lot size according to cost. There are two main factors that affect the cost: equipment adjustment cost and inventory keeping cost [4]. The larger the batch size, the less the equipment adjustment cost and the less the adjustment cost allocated to each product; However, the storage cost will increase accordingly; On the contrary, the adjustment cost of small unit products in batches will be large, and the storage cost will be reduced accordingly. The principle of seeking economic batches is to use mathematical methods to obtain the sum of these two costs as the smallest batch, which is both economic batch. As shown in Figure 1:

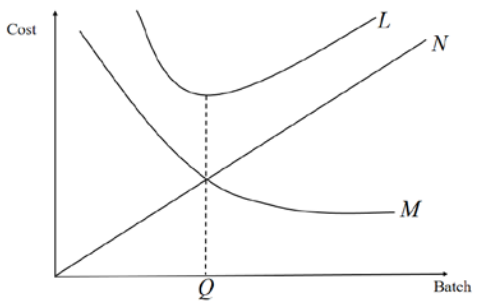

Fig.1 Economic Lot Size

In fig. 1, line M is the adjustment cost curve, line N is the storage cost curve, and line L is the sum of the above two costs. when the sum of the above two costs is the smallest, the corresponding batch Q is the economic batch.

The annual equipment adjustment cost can be expressed by the following formula:

$$Cost_{Annual\ equipment\ adjustment} = A \times \frac{N}{Q} \quad (2)$$

In which: A is the cost of each equipment adjustment, N is the annual output, and Q is the batch.

Inventory keeping costs can be expressed by the following formula:

$$Cost_{Inventory\ keeping} = C \times \frac{Q}{2} \quad (3)$$

Where: C is the average storage cost per unit product

Total cost Y is the sum of two costs:

$$Y = A \times \frac{N}{Q} + C \times \frac{Q}{2} \quad (4)$$

When $dy/dp=0$, the cost is the smallest, so,

$$A\times\left(-\frac{N}{Q^2}\right)+\frac{1}{2}C=0$$

$$Q=\sqrt{\frac{2NA}{C}} \qquad (5)$$

This formula is the formula for calculating the economic lot size.

3. Logistic Equation

The logistic equation is a nonlinear differential equation, and its mathematical model belongs to a continuous, monotonically increasing S -shaped curve with single parameter k as the upper asymptote. As we all know, there are a lot of S -type changes in economics, and the Logistic equation is a mathematical model that can describe such changes. It is characterized by slow growth at the beginning, rapid growth in the middle stage, and declining and stabilizing growth in the future.

In economics, if the basic characteristics of the problem are [5-6]:

When time t is very small, it grows exponentially; However, when t is increasing, the growth rate is decreasing, and it is getting closer and closer to a certain value, we can consider using the Logistic equation to solve it.

Some economic problems, such as the development of new products in the market, can be well analyzed by using the idea of logistic equation. According to the logistic equation, we can build a new product promotion model by establishing a mathematical model.

For example, when a new product comes out, the sales volume at t moment is $f(t)$. Because the product is a new product and there is no alternative product, the growth rate of product sales at t moment dx/dt is directly proportional to $f(x)$.

At the same time, there is a certain market capacity N in the sales volume of products, and statistics show that dx/dt is also proportional to the number of potential customers $N-f(x)$ who have not yet purchased this new product, so there is a model that conforms to the logistic equation, so there is a general solution:

$$\frac{dx}{dt}=kx(N-x)\ \frac{dx}{dt}=kx(N-x) \qquad (6)$$

In which k is the proportional coefficient. by separating the integral of variables, we can get:

$$x(t)=\frac{N}{1+Ce^{-kNt}} \qquad (7)$$

By the following formula:

$$\frac{dx}{dt}=\frac{CN^2ke^{-kNt}}{\left(1+Ce^{-kNt}\right)}$$

$$\frac{dx^2}{dt^2}=\frac{Ck^2N_3e^{-kNt}\left(Ce^{-kNt}-1\right)}{\left(1+Ce^{-kNt}\right)^2} \qquad (8)$$

When $x(t^*)<N$, there is $dx/dt>0$, that is, the sales volume $x(t)$ increases monotonously.

When $x(t^*)=N/2$, $x(t^*)=N/2$;

When $x(t^*)>N/2$, $dx^2/dt^2<0$;

When $x(t^*) < N/2$, $dx^2/dt^2 > 0$; That is, when the sales volume is more than half of the demand, the products sell best. When the sales volume is less than half, the sales speed will increase continuously. Similarly, when the sales volume reaches half, the sales speed will decrease continuously.

The sales curves of many products are very similar to those of the Logistic equation [7]. Therefore, analysts believe that the products should be produced in small quantities at the initial stage of launch. When product users are between 20% and 80%, products should be produced in large quantities, but when product users exceed 80%, enterprises should develop new products.

4. Optimization of Dynamic System of Economic Management

4.1 Analysis of Maximum Analysis Chart of Economic Target Product

The maximum analysis graph of economic target product is a widely used method to study the maximization of economic system, and its geometric figures can be described by two types as shown in Figure 2.

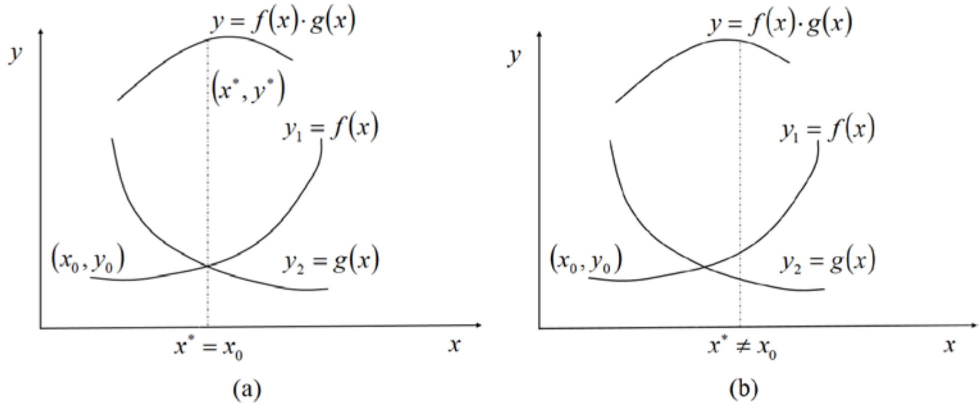

Fig.2 Maximum Analysis Chart of Economic Target Product

The common feature of the two types of graphs is that a product function $f(x) \cdot g(x)$ is composed of a monotone increasing function $f(x)$ and a monotone decreasing function $g(x)$. The relationship between economic objective function and management variables is intuitively and qualitatively reflected by mathematical methods, and the changing state of economic objective function with the increase or decrease of management variables and the system state after the increase or decrease of management variables are vividly described.

The main difference between these two types of graphs is: in Fig. 2(a), the maximum point (x^*, y^*) of the product function and the intersection point (x_0, y_0) of the two functions have $x^* = x_0$; while in Fig. 2(b), the maximum point (x^*, y^*) of the product function and There is $x^* \neq x_0$ at the intersection of two functions (x_0, y_0).

In the quantitative analysis and calculation of the maximum analysis chart of economic target product, we are mainly concerned about how to get the maximum value, and how to get the value of management variables in the state of maximum economic target, so that we can maximize the economic target by selecting the best management variables. Therefore, in the analysis of maximizing economic target product, how to quantify and define qualitative analysis and how to simplify quantitative calculation are the key to make the analysis of maximizing economic target product correct and widely used. Through the above analysis, we think that the situation in Figure 2(a), where the intersection points of two curves coincide with the poles of the product function, is very satisfactory, because it can transform the problem of the maximum value of the product function we want into the problem of solving the intersection points of two curves, which is easier to solve.

4.2 Analysis of Advertising Effect

The information society makes advertising a powerful means to adjust the sales of goods. What is the internal relationship between advertising and sales? How to evaluate the advertising effect in different periods. This also needs to be studied with the help of mathematical models. First of all, it is considered that advertising can directly promote the sales speed of products. Taking the sales speed as the research object, let $s(t)$ be the sales speed of products at time t, and make the following assumptions:

Without considering the role of advertising, the sales speed has the nature of natural attenuation, that is, the sales speed of products decreases with time, which satisfies the sales speed of this nature:

$$\frac{ds}{dt} = -\lambda s(t) \tag{9}$$

In which λ is the attenuation factor.

The sales speed of products will increase due to advertising, but there is a certain limit to the increase. When the products are saturated in the market, the sales speed will tend to the limit value. At this time, no matter what form of advertising is adopted (excluding other promotional means), the sales speed cannot be increased.

The sales speed of products is related to the level of advertising investment. Let $A(t)$ be the level of advertising investment per unit time at time t (expressed in terms of cost), and p be the response coefficient of investment, that is, the influence of investment $A(t)$ on sales speed. According to the above assumptions, there are:

$$\frac{ds}{dt} = pA(t)\left[1 - \frac{s(t)}{M}\right] - \lambda s(t) \tag{10}$$

The first item at the right end of the above formula reflects the influence of advertising investment on sales speed. $\left[1 - \frac{s(t)}{M}\right]$ is equivalent to a switch function. Obviously, when $A(t) = 0$ or $s = M$, there are:

$$\frac{ds}{dt} = \lambda s(t) \tag{11}$$

The second item at the right end of formula (10) shows the characteristics of natural decline in sales speed. To determine the form of $A(t)$, it is assumed that the following advertising strategies are selected:

$$A(t) = \begin{cases} A, & 0 < t < \tau \\ 0, & t \geq \tau \end{cases} \tag{12}$$

That is, in time τ, an average investment of constant A is made to advertise, and equation (10) is solved under this condition. In time period $(0, \tau)$, assuming that the total investment for advertising is a, the investment per unit time is:

$$A = \frac{a}{\tau} \quad (13)$$

Substituting into formula (10), the arrangement is as follows:

$$\frac{ds}{dt} = \left(\lambda + \frac{p}{M}\frac{a}{\tau}\right)s = p\frac{a}{\tau} \quad (14)$$

Make

$$\lambda + \frac{p}{M}\frac{a}{\tau} = b, \frac{pa}{\tau} - c \quad (15)$$

There are

$$\frac{ds}{dt} + bs = c \quad (16)$$

The general solution is:

$$s(t) = C_1 e^{-bt} + \frac{c}{b} \quad (17)$$

In which C_1 is an integral constant.

If the initial sales speed is $s(0) = s_0$, then:

$$s(t) = s\frac{c}{b}\left(1 - e^{-bt} + s_0 e^{-bt}\right), 0 < t < \tau \quad (18)$$

When $t \geq \tau$, according to equation (12), A= 0, then equation (10) degenerates into $\frac{ds}{dt} = \text{-}\lambda s(t)$: its solution is:

$$s(t) = s(\tau)e^{\lambda(r-1), t \geq \tau} \quad (19)$$

Synthesizing (18) and (19), under the condition of (12), the solution of product sales speed advertising model can be written as:

$$s(t) = \begin{cases} \frac{c}{b}\left(1 - e^{-bt}\right) + s_0 e^{-bt}, & 0 < t < \tau \\ s(\tau)e^{\lambda(\tau-1)} & t \geq \tau \end{cases} \quad (20)$$

The change of sales speed with time is shown in Figure 3.

Fig.3 Changes of Sales Speed with Time

4.3 Market Dynamic Equilibrium Price

Ordinary differential equation is the simplest and most important system of equations in algebraic system. Ordinary differential equation is a particularly important and frequently used tool when solving practical social life problems, especially economic problems. Ordinary differential equations in cost and profit are simple and easy to understand, but ordinary differential equations have changed the calculation method in the general sense. With the ability of computer operation, manpower can be completed for a long time in a short time, and sometimes it can be a few months' workload. These facts all prove that scientific and technological forces can make great contributions to business.

Ordinary differential equations have a wide range of functions. As we mentioned earlier, many aspects such as aerospace, electronic communication, chemistry and so on all need to use ordinary differential equations to solve the problems encountered in the actual research process. Studying the new solvable types of specific ordinary differential equations can help people deal with some difficult problems accurately in different disciplines, which is the main channel to solve difficulties. Therefore, we should do a deeper research on the new solvable types of ordinary differential equations, and use the solutions of equations to promote the rapid development of different disciplines.

Let market price $P = p(t)$, demand function $Q_d = b - ap(a,b>0)$, supply function $Q_s = -d + cp(c,d>0)$, and let the rate of change of price P with time t be proportional to excess demand $Q_d - Q_s$, and find the price function $P = p(t)$.

By the meaning of the question, there are:

$$\begin{cases} \dfrac{dP}{dt} = A\cdot(Q_d - Q_s) = A\cdot(a+c)p + A(b+d) \\ p|_{t=0} = p(0) \end{cases} \quad (21)$$

According to the general solution formula of the first-order linear equation:

$$P = e^{-\int A(a+c)dt}\left(\int A(b+d)\cdot e^{\int A(a+c)dt} + C_1\right) = \frac{b+d}{a+c} + C_1\cdot e^{-A(a+c)\cdot t} \quad (22)$$

From the initial condition $t=0, p = p(0)$, we get:

$$C_1 = p(0) - \frac{b+d}{a+c} \quad (23)$$

Substituting into the above formula:

$$P = \left(p(0) - \frac{b+d}{a+c}\right)e^{-A(a+c)\cdot t} + \frac{b+d}{a+c} \quad (24)$$

Obviously, when:

$$\lim_{t\to\infty} p(t) = \lim_{t\to\infty}\left[\left(p(0) - \frac{b+d}{a+c}\right)e^{-A(a+c)\cdot t} + \frac{b+d}{a+c}\right] = \frac{b+d}{a+c} \quad (25)$$

That is, when $t \to \infty$, the equilibrium price is $\dfrac{b+d}{a+c}$.

5. Conclusion

Differential equations have many functions, especially in economic management. It must be studied quantitatively by economics. And the quantitative research of higher mathematics is one of the most important and basic mathematical tools in economic management. Exploring the optimization of economic management dynamic system is the purpose of economic management, but restricting the optimization goal of economic management dynamic system is not a single factor. There are often contradictory variables and factors, which are the dynamic unity of quantitative change and qualitative change. Therefore, many scholars are studying the optimization of economic management dynamic system. With the development of social economy, higher mathematics will not only be an effective tool to solve problems in economic management, but also be more and more applied in other fields, such as environmental governance, population prediction, the spread of infectious diseases, the distribution of drugs in human body, etc., to solve more and more practical problems for people.

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