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Research on Deep Neural Network Solution of Ordinary Differential Equations and Its Application

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Abstract: This article discusses the application of a single hidden layer neural network algorithm in solving numerical ordinary differential equations. Set the approximate solution with neural network structure to meet the initial boundary value conditions of ordinary differential equations, and discretize the weights of neural networks in the original equations. The optimization problem is transformed into the training problem of the neural network in the approximate solution, so that the approximate solution is close to the real solution. The changes in the structure, weights, and thresholds of the neural network are observed through network deduction. Numerical examples based on the python language prove the algorithm's performance Effectiveness.

1. Introduction

In scientific computing and engineering applications, it is often necessary to solve differential equations. However, analytical methods can only be used to solve certain special types of equations. In other cases, numerical solutions are mainly used. For ordinary differential equations, the commonly used difference method is Euler's method, Improved Euler’s method and Heun’s method, etc. Among them, Euler’s method is the simplest one-step method, which has a small amount of calculation but low accuracy. The improved Euler method adds a correction process, The accuracy of the solution has been greatly improved. The Heun's method is further improved on the basis of the Euler method, with higher accuracy. In recent years, neural networks have achieved remarkable results in various fields such as machine learning, deep learning, and artificial intelligence. The application of neural networks to solve differential equations has become a hot topic. Literature uses cosine-based neural networks to solve ordinary differential equations. Literature uses neural networks and wavelet analysis to solve differential equations. In 1988, Lagaris and Likas proposed a neural network algorithm for solving the initial boundary value problem of partial differential equations. Since a single hidden layer neural network can approximate any continuous function with arbitrary precision, the algorithm can be used to solve differential equation problems. 2010 Baymani, Kerayechia, EffatiS used neural network algorithms to solve the stokes equation. In 2017, Raissi, Perdikaris, and Kar niadakis proposed a neural network algorithm based on Gaussian processes. Long, Lu and Ma proposed based on convolutional differential A neural network algorithm for solving partial differential equations by operators. In 2018, Han, Jentzen and others proposed an algorithm for solving high-dimensional backward differential equations using a multilayer neural network. In 2019, He and Xu the relationship between convolutional neural network and multigrid is discussed. The neural network algorithm used in this paper takes the residual of ordinary differential equations and the sum of initial boundary value conditional residuals as the loss function, by minimizing the loss function Solve and evaluate the model to train the weights in the neural network to obtain an approximate solution of the required points. Based on the python language, the Adam optimizer solves the discrete optimization problem.

2. Basic Theory

In 1943, Mcculloch and Pitts abstracted the biological neuron model as the artificial neuron model shown in Fig. 1. Suppose the input of the neuron is x1, x2,..., xn, these inputs are passed with weights w1, w2 ,..., wn is connected to transfer, and then the total input is obtained by weighted summation, and then compared with the neuron threshold b, and finally the activation function σ is used to perform nonlinear changes to obtain the neuron output y. Artificial neuron structure model As shown in Figure 1:

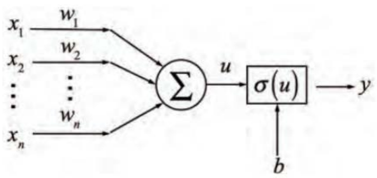

Fig.1 Structural Model Ofartificial Neuron

The mathematical model of the above neuron working process is as follows:

$$\begin{cases} u = \sum_{i=1}^{n} w_i x_i \\ y = \sigma(u - b) \end{cases}.$$

Weight, linear summation and activation function are the three basic elements of neuron. Weight and threshold are the basic parameters of artificial neuron model. For the selection of activation function, the most ideal activation function is shown in Figure 2(a) Step function:

$$sgn(x) = \begin{cases} 1, x \geqslant 0 \\ 0, x < 0 \end{cases}.$$

The step function maps the input of the neuron to 0 or 1, respectively representing the two states of “inhibition” and “excitement” of the neuron. However, due to the discontinuity and non-direction of the step function at the step point, practical applications The Sigmoid function shown in Figure 2(b) is often used in:

$$sigmoid = \frac{1}{1 + e^{-ax}}, a > 0,$$

As the activation function, Sigmoid can continuously map the input value of the neuron to the range of (0,1), and can perform multiple derivation, which conforms to the characteristics of probability distribution

3. Algorithm Ideas for Neural Network to Solve Ordinary Differential Equations

In practical problems, the expected value or the true value is often not informed, and the definition of the artificial neural network error e needs to be adjusted accordingly. The general n-th order ordinary differential equation has the form:

$$F\left(x, y, \frac{dy}{dx}, \cdots, \frac{d^n y}{dx^n}\right) = 0,$$

Here F x, y, dydx,..., dnydx (n) are known functions of x, y, dydx,..., dnydxn, and must contain dnydxn; y is an unknown function, and x is an independent variable. Consider the following The initial-boundary value problem of ordinary differential equations: Solve y to satisfy:

$$\begin{cases} F\left(x, y, \frac{dy}{dx}, \cdots, \frac{d^n y}{dx^n}\right) = 0, & x \in [c, d] \\ y(c) = g. \end{cases}$$

Set the approximate solution y containing the neural network structure to replace the real solution y in the ordinary differential equation, then the problem of solving the original equation is transformed into a discrete optimization problem about the weights of the neural network. The loss function Loss is defined as:

$$Loss = \frac{1}{n}\left(F\left(\tilde{x}, \tilde{y}, \frac{d\tilde{y}}{d\tilde{x}}, \cdots, \frac{d^n \tilde{y}}{d\tilde{x}^n}\right)\right)^2 + (\tilde{y}(c) - g)^2,$$

When the loss function approaches zero, the approximate solution y obtained through the neural network approximates the real solution y. Therefore, the loss function can be used to replace the error value in the back propagation of the error. Suppose the approximate solution y of ordinary differential equations has Single hidden layer neural network structure, where the input layer is x =c,x1,...,x (n-2,d) T, the hidden layer contains m neurons, and its weights are Wi = wi1,wi2,..., win (), the threshold is bi = b1, b2,..., bm (), where i = 1, 2,..., m. The weight of the output layer is Wi = wi1, wi2,..., win (), i = m + 1, m + 2,..., 2m. Using the signal forward propagation and error back propagation process in the BP algorithm, the weights and thresholds of each neuron are constantly adjusted to reduce the loss function to the specified range. The approximate solution y of the ordinary differential equation can be obtained.

4. Application Research of Ordinary Differential Equations

We need different mathematical models to correspond to different computing objects, so that data loss and confusion will not occur in the process of more complex calculations. At the same time, the simplification process of the modeling purpose of the research object must be completed. Secondly, it is necessary to find the key to solving the problem in other similar models, that is, similar problem-solving laws or ideas. This kind of operation can achieve effective feedback on its results, so that the correct answer to the actual problem can be obtained. Of course, mathematical modeling itself is a process of using creative thinking to analyze and solve problems. Therefore, it is necessary to find the right entry point to solve the problem well in combination with ordinary differential equations, so as to fully reflect the problem-solving strategy of modeling thinking. The problem-solving ideas of ordinary differential equations will bring a new problem-solving experience to our abstract thinking. It can express various complex relationships in mathematical models with the most condensed mathematical operations. Although the whole derivation process is very cumbersome, the use of this way of solving problems and thinking, combined with the use of mathematical modeling methods, will bring many unexpected benefits to the solution of mathematical problems. Now people often combine ordinary differential equations with mathematical modeling methods. This will realize the simplification of complex real-world problems, and the application of ordinary differential equations in mathematical modeling also makes mathematical science enter a stage of rapid development.

In economics and management, which are closely related to mathematics, it is often necessary to discuss practical issues such as the change and growth of the variables studied. It is necessary to combine ordinary differential equations with mathematical modeling, and formulate a summary of the law of change. For example, when a company promotes a new type of product to the market, the number of sales at T is Y(T), and the product has won a good reputation and sales are good. Then, the increase in product sales dT/dY is proportional to Y(T) at the time of product T, and the sales volume of this product will also reach saturation. The market's maximum consumption M, according to the company's survey results, dT/Y(T) is also directly proportional to the potential sales volume of this product. Therefore, we can get dY/dT=kY(M-Y), where the constant k is a number greater than zero. After further simplification, we can get y(t)=m/(1+Ce-kmt). And by dy/dt=cm2ke-kmt/(1+Ce-kmt) and d2y/dt2=ck2m3e-kmt(Ce-kmt-1)/(1+Ce-kmt)3, then if y(t*) is Between zero and m, if dt/dy is greater than 0, there will be an increase in sales, and vice versa.

Ventilation problems refer to certain factories, such as chemical factories, that produce a lot of toxic and harmful gases during the production process. If these toxic and harmful gases cannot be discharged from the factory workshop in time, it will cause great harm to the employees in the workshop. To ensure sufficient oxygen injection is a ventilation problem faced by chemical plants. For example, in a factory workshop of 10,000 cubic meters, the content of carbon dioxide in the air is 12 parts per 10,000, and the content of fresh air is 4 parts per 10,000. If it needs to be less than six ten-thousandths after ten minutes, how much fresh air should be injected every minute? According to the specific relational formula: Inlet volume (Exhaust volume) = Inlet speed * Gas concentration * Time used, the actual increase in gas = Injection volume-Exhaust volume. According to the actual situation and combining the ideas of ordinary differential equations and mathematical modeling methods, it can be assumed that at a certain time s, the total amount of carbon dioxide is wx, and at another time s+ds is w(x+dx), so use Ordinary differential equations can get the increment as wdx, that is, wdx=agds-axds, after simplification, dx/(xg)=-ads/w can be obtained. After calculation, a=-1080ln0.25 is obtained, which is the volume flow rate a=1500m3/min. In other words, 1500 cubic meters of fresh air per minute can reduce the carbon dioxide content in the factory floor to less than six ten thousandths after ten minutes.

5. Conclusion

This paper discusses the application of a single hidden layer neural network algorithm in solving numerical ordinary differential equations. The paper links the problem of solving differential equations with the problem of optimizing the loss function, and training the neural network with the problem of optimizing the loss function to obtain The approximate solution of the differential equation, and the changes of the neural network weight and the threshold value are observed through network deduction. Finally, the numerical example solved by the python language proves the accuracy and practicability of the neural network algorithm. Through the above analysis, we can see the huge advantages of ordinary differential equations in mathematical modeling applications. It plays a great role in the full use of modeling thinking and the simplification of complex mathematical problems. Therefore, in the future development of mathematics, we must grasp the application of ordinary differential equations in mathematical modeling in order to expand the development in the field of mathematics.

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