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Application of Wavelet Analysis in Signal Processing

Yuantang Duan, Shiyuan Zhu, Hongliang Zheng

School of Electronic Information Engineering, Beijing Jiaotong University, Beijing 100044, China

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Abstract: Signal analysis is a significant part of current information processing. In the early days, when the Fourier transform was mainly used for processing, only frequency domain analysis could be performed, resulting in incomplete signal analysis. Based on this, the wavelet transform introduces a time-domain window, which makes signal analysis more effective. Wavelet analysis is the inheritance and development of Fourier transform. This paper introduces the actual background of wavelet theory and the basic principles of wavelet transform. On the basis of the comparative analysis of Fourier transform and wavelet transform, it focuses on the application of Matlab wavelet analysis in speech signal analysis, denoising and compression.

1. Introduction

Wavelet transform theory is a new theory developed in recent decades. It is an effective harmonic analysis tool based on Fourier transform. As the earliest harmonic analysis tool, Fourier analysis plays a very significant role in signal spectrum analysis and is an important branch of harmonic analysis. Fourier transform plays an extremely significant role in signal analysis, but there are also shortcomings. When it is necessary to analyze the frequency components of a signal at a certain time point or a certain period of time, the traditional Fourier transform is useless. Therefore, on this basis, the short-time Fourier transform, Gabor transform and the revolutionary wavelet transform were developed. Wavelet transform is to transform the signal by using the analysis window with fixed area but changing shape. Its multi-resolution analysis is very suitable to analyze non-stationary signals. With the development of the times and the progress of science and technology, the theory of wavelet transform is put forward through the efforts of generations of researchers. This theory provides an adaptive time-frequency window structure. In practical application, this structure meets the requirements of narrow frequency window in the low frequency part and wide frequency window in the high frequency part, making the time-frequency analysis method further developed. The development of wavelet analysis has experienced a long historical process. Its development history can be traced back to Haar’s work in 1909. At that time, the development of wavelet analysis was very slow. It was not until the research of Grossmann and Morlet in 1980 that the rudiment of modern wavelet analysis was formed. Since 1986, due to the work of Y. Meyer, S. Mallat and I. Daubechies, wavelet analysis has developed rapidly, forming a new discipline and producing many valuable applications. Because wavelet analysis method has higher frequency resolution and lower time resolution in low frequency part, and higher time resolution and lower frequency resolution in high frequency part, it is called “mathematical microscope" for signal analysis. It is because of the "microscope" characteristic of wavelet analysis that wavelet analysis has the adaptability to the signal. It also makes up for the shortcomings of various transforms in the Fourier transform system. Therefore, wavelet analysis technology has been widely used and become the focus of many disciplines. At present, wavelet analysis has become a very active research field in the world. It has been widely used in signal processing, image processing, speech analysis, seismic exploration, atmospheric turbulence and scale analysis, system identification and spectrum estimation, pattern recognition, quantum physics and many nonlinear sciences.

As a new time-frequency analysis method, wavelet analysis realizes the perfect structure among universal function, Fourier analysis, harmonic analysis and numerical analysis, and becomes another important signal analysis method after Fourier transform analysis method. It has been studied by many researchers and has been greatly expanded in theory and application. Based on this, this paper focuses on the application of wavelet analysis in signal processing.

2. Wavelet Transform Theory

2.1 Concept of Wavelet

The so-called wavelet, simply speaking, is a family of functions generated by translation and scaling of a function satisfying the condition $\int R\varphi(t)dt = 0$, that is, $\varphi a, b(t)=|a|^{-1/2}\varphi(\frac{t-b}{a})$,a,b∈R,a≠0.

Given an energy limited signal $f(t)$, the continuous wavelet transform (CWT) of $f(t) \in L^2(R)$ is defined as:

$W_{f(a,b)}=\int_{-\infty}^{+\infty} f(t)\varphi * a, b(t)dt, a \neq 0$,$\varphi^*$ is a conjugate of wavelet function φ. When φ(t) meets the following conditions:

$$C_\varphi = \int_{+\infty}^{-\infty} \frac{|\varphi(\omega)|^2}{|\omega|} d\omega < +\infty$$

It is ideal to recover to $f(t) = \frac{1}{C_\varphi} = \iint R \frac{W_{f(a,b)\varphi * a,b(t)}}{a^2} dadb$

Because of the redundancy of continuous wavelet transform, its parameters need to be discretized in practical application. The physical meaning of wavelet can be reflected from $\int R\varphi(t)dt = 0$. $W_{f(a,b)}$ can be described as the filtering of signal (function) $f(t) \in L^2(R)$ through band-pass filter, so the process of wavelet transform can be regarded as filtering.

Because the information of one-dimensional signal x(t) is redundant after it is transformed into two-dimensional $W_{f(a,b)}$) by wavelet transform, so discrete wavelet transform is commonly used in engineering applications. The current method is to discretize the scale power series, discretize the scale factor a and the shift factor b. If the scale factor a is discretized in a binary way, the dyadic wavelet and dyadic wavelet transform are obtained.

2.2 Time Frequency Limitation of Wavelet Transform

Wavelet function φ(t) satisfies formula $\int R\varphi(t)dt = 0$, showing that φ(t) has oscillation characteristics. Its property reflects some frequency characteristics of wavelet function φ. The oscillation of $\varphi a, b(t)$ increases with the increase of $\frac{1}{|a|}$, (a is frequency parameter, b is time-domain parameter). In practice, φ is always a function with compact support or fast attenuation, that is, a function with local time and frequency, so wavelet transform can also implement time-frequency localization of signal, but the localization mode of wavelet transform and STFT transform is obviously different. The time-frequency localization of wavelet transform is closely related to the frequency level. In high frequency region, the degree of time localization is high. In low frequency region, frequency localization is high, so it has better time-frequency resolution. Because of the redundancy of continuous wavelet transform, discrete wavelet transform (DWT) is often used to sample (a, b) in a certain way. The sampling step is small for small-scale high-frequency components, but long for large-scale low-frequency components.

2.3 Origin and Function of Wavelet Transform

Wavelet analysis method is a kind of time-frequency localization analysis method with fixed window size but changeable shape, and changeable time window and frequency window, that is, it has higher frequency resolution and lower time resolution in low frequency part, and higher time resolution and lower frequency resolution in high frequency part. It is this characteristic that makes wavelet transform adaptive to signals.

The time-frequency window characteristic of wavelet transform only affects the position of the window on the phase plane axis, so the sampling step of wavelet transform for different frequencies in the time domain can be adjusted, which is in line with the characteristics of slow change of low frequency signal and rapid change of high frequency signal, better than the classical Fourier transform and STFT. Generally speaking, wavelet transform has better time-frequency window characteristics than STFT.

Therefore, wavelet transform has the following characteristics and functions.

Firstly, it has the characteristics of multi-resolution, and can observe the signal step by step from coarse to fine.

Secondly, the wavelet transform can be regarded as signal filter with different scale a by band-pass filter with the basic frequency characteristics (ω).

Thirdly, the basic wavelets are selected properly to make the φ(t) be a finite support in time domain. φ(ω) is also concentrated in frequency domain. At this time, wavelet has the ability to represent the local characteristics of the signal in both time and frequency domains, which is conducive to detect the transient or singular points of the signal.

2.4 Comparison of Fourier Transform and Wavelet Transform

Firstly, the essence of Fourier transform is to decompose the energy-limited signal f (t) into the space that takes $\{e^{j\omega t}\}$ as the orthogonal basis. The essence of wavelet transform is to decompose into $L^2(R)$ (square integrable function) space.

Secondly, the basic functions used in Fourier transform are unique, only $\sin\omega t$, $\cos\omega t$ and $e^{j\omega t}$. The wavelet function used in wavelet analysis is not unique. Sometimes the results of different wavelet functions are different in the same engineering problem.

Thirdly, wavelet transform has good localization ability in time and frequency domain. But Fourier transform can only determine the signals with easy components in frequency domain, and Fourier transform is easy to express the signal as the sum of the frequency components. But there is no localization ability in time domain.

Fourthly, the bandwidth Δf of the wavelet transform band-pass filter is proportional to the central frequency f, and its value is a constant, that is, the filter has a constant relative bandwidth. The bandwidth of the band-pass filter of short-time Fourier transform is independent of the central frequency f.

2.5 Wavelet Packet Transform

Wavelet packet transform, also known as wavelet packet decomposition, is one of the wavelet analysis methods, which realizes the optimization of wavelet analysis. The wavelet packet transform describes the signal more finely, and divides the frequency band into multiple levels, thereby realizing the effective decomposition of the low-frequency part and the high-frequency part of the signal. In addition, due to the adaptive selection of scales, it is adaptable in the selection of frequency bands, which can improve the registration of the frequency spectrum, thereby improving the resolution of the time-frequency analysis. In addition, the corresponding decoding and encoding process of wavelet packet transform has no redundancy and lack, so it is an optimized embodiment of wavelet analysis. This section presents a schematic diagram of the decomposition of the 3-layer wavelet packet of the signal S.

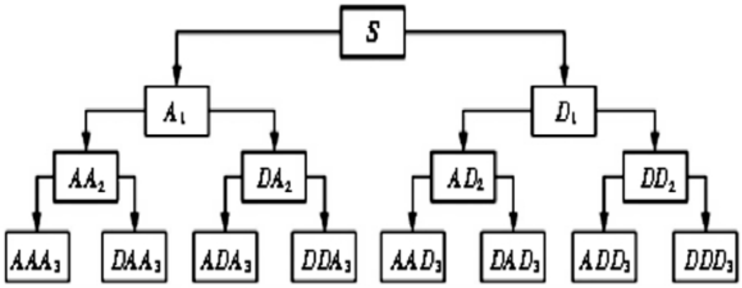

Fig.1 Schematic Diagram of Three-Layer Wavelet Packet Decomposition of Signal s

3. Application of Matlab Wavelet Transform in Signal Processing

As a means of signal processing, wavelet transform is gradually valued and applied by more and more workers in many fields. It has achieved remarkable results in many applications. Compared with the traditional processing method, wavelet transform has a qualitative leap and has greater advantages in signal processing. Its typical applications include signal noise reduction and compression, analysis of common signals and detection of signal characteristics.

3.1 Application of Wavelet Transform in Signal Denoising and Compression

Signal denoising and compression is one of the important applications of wavelet. Wavelet denoising is mainly based on three characteristics of wavelet transform.

The first is multi-resolution characteristics. Due to the adoption of multi-resolution method, the nonstationarity of signal can be well characterized, such as abrupt change and breakpoint, etc., and the noise can be eliminated according to the distribution of signal and noise at different resolutions.

The second is decorrelation. Wavelet transform can decorrelate the signal, and the noise tends to whiten after transform, so wavelet domain is more conducive to denoising than time domain.

The third is flexible selection of basis function. Wavelet transform can flexibly select the basis function, and can also select multi-band wavelet and wavelet transform according to signal characteristics and denoising requirements. For different occasions, different wavelet generating functions can be selected.

3.1.1 Wavelet Enhancement of Speech Signal

The auditory range of the human ear to the sound of different intensity and frequency is called the sound field. In the sound field of the human ear, the subjective perception of sound auditory psychology mainly includes loudness, pitch, timbre, masking effect, high-frequency positioning and other characteristics. Among them, loudness, pitch and timbre can be subjectively used to describe any complex sound with three physical quantities of amplitude, frequency and phase, which are called the “three elements” of sound. In the case of multiple sound sources, the masking effect of human ear is more important and it is the basis of psychoacoustics. The wavelet enhancement of speech signal is the process of denoising the signal. In fact, it is the process of suppressing the useless part and enhancing the useful part of the signal. There are many wavelet denoising methods, which can be summarized as shielding denoising method, threshold denoising method, modulus maximum detection denoising method and so on. Generally, the process of one-dimensional signal denoising can be divided into the following three steps.

The first step is wavelet decomposition of one-dimensional signal. Select a wavelet and determine the decomposition level, and then carry out the decomposition calculation.

The second step is threshold quantization of wavelet decomposition high frequency coefficients. Select a threshold value to quantify the high frequency coefficients of each decomposition scale.

The third step is one-dimensional wavelet reconstruction. Carry out one-dimensional wavelet reconstruction according to the lowest low-frequency coefficients and high-frequency coefficients of wavelet decomposition.

3.1.2 Threshold Selection for Wavelet Enhancement of Speech Signal

Whether using wavelet transform to enhance or compress speech signal, the selection of threshold is the key. When compressing, if the threshold is too large, the signal will be distorted. If the threshold is too small, the compression is not significant. When denoising, if the threshold is too large, although it can reduce the noise in the signal, it will remove part of the energy of the signal, and the reconstructed signal will also have large distortion. If the threshold is too small, the reconstructed signal will contain too many noise components, and fail to achieve the purpose of denoising.

3.1.3 Simulation Results and Analysis of Speech Signal Denoising Based on Wavelet

In this paper, wavelet denoising is used to eliminate the noise of one-dimensional signal. It can be seen from the image that the enhanced speech signal is very smooth, basically without noise components, showing a lot of information of the original signal, but also removing some details of the original signal. In order to improve the denoising effect, the improvement effect is not obvious by changing the wavelet basis or increasing the number of layers, so the traditional wavelet denoising algorithm still has a lot of room for improvement, but its future is promising. Comparison of Signal with Original Signal, Noise and Denoising Signal (From top to bottom) is shown in Figure 2.

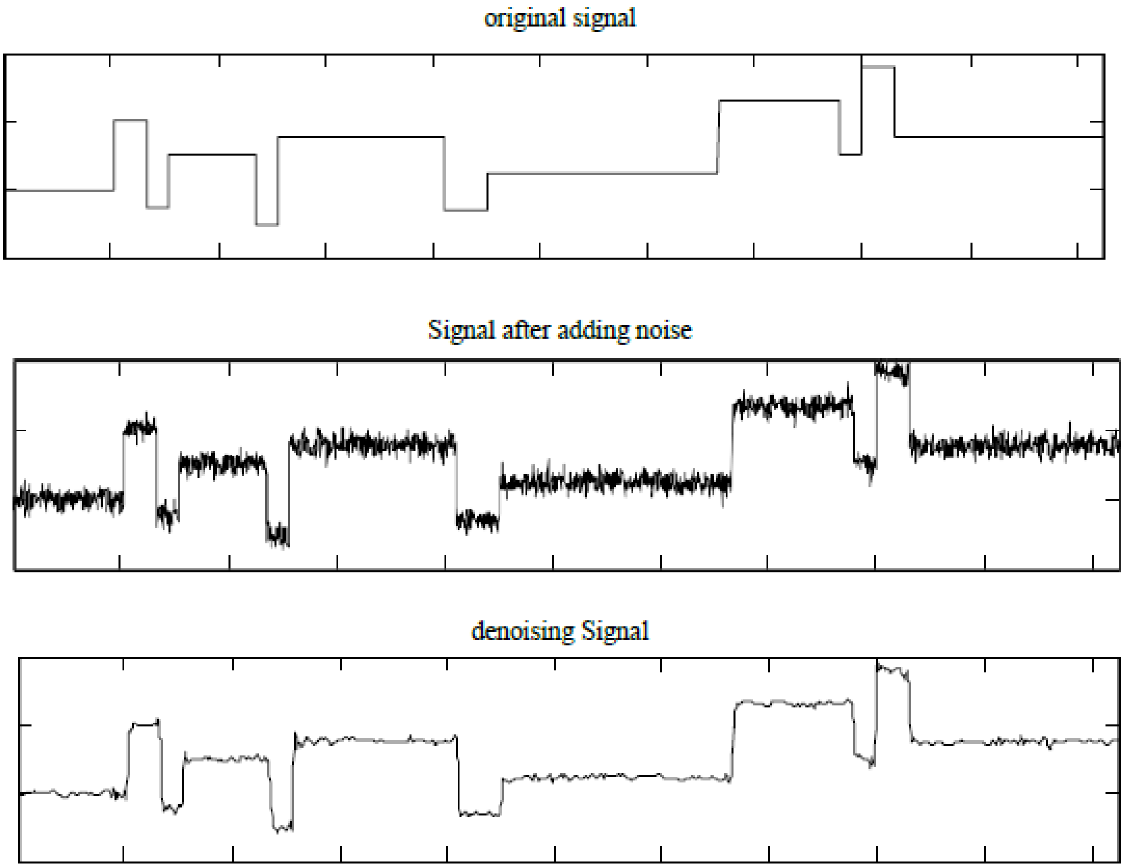

Fig.2 Ure 2 Comparison of Signal with Original Signal, Noise and Denoising Signal (from Top to Bottom)

3.2 Application of Wavelet Analysis in Speech Signal Compression Coding

3.2.1 Wavelet Compression of Speech Signal

The basic purpose of signal compression is to reduce the cost of signal storage as much as possible without losing the information carried by the signal. The basic idea of signal compression based on wavelet transform is that after wavelet transform, the detail coefficients which are not the main properties of the signal are filtered by the scope value, and then the original signal is recovered by using the approximate coefficients of low sampling rate and a small amount of detail coefficients concerned, so as to realize the compression of the signal.

3.2.2 Application of Wavelet Analysis in Speech Signal Compression and Coding

Compression coding is an important part of speech signal processing, and its focus is to reduce the speech signal coding rate. At present, the methods are mainly divided into waveform coding, parameter coding, and hybrid coding. Waveform coding has simple design and high precision, but the digital rate is high and the compression efficiency is low. Although parameter encoding can significantly reduce the digital rate, it is highly complex and difficult to guarantee accuracy. Hybrid coding combines the advantages of the above two types of methods, effectively improving the digital rate and accuracy. The coding method based on wavelet transform is essentially a transform domain coding method, which mainly discretizes and orthogonalizes the input signal, and then quantizes the coding coefficients for compression. A typical wavelet transform coding block diagram is shown in Figure 3.

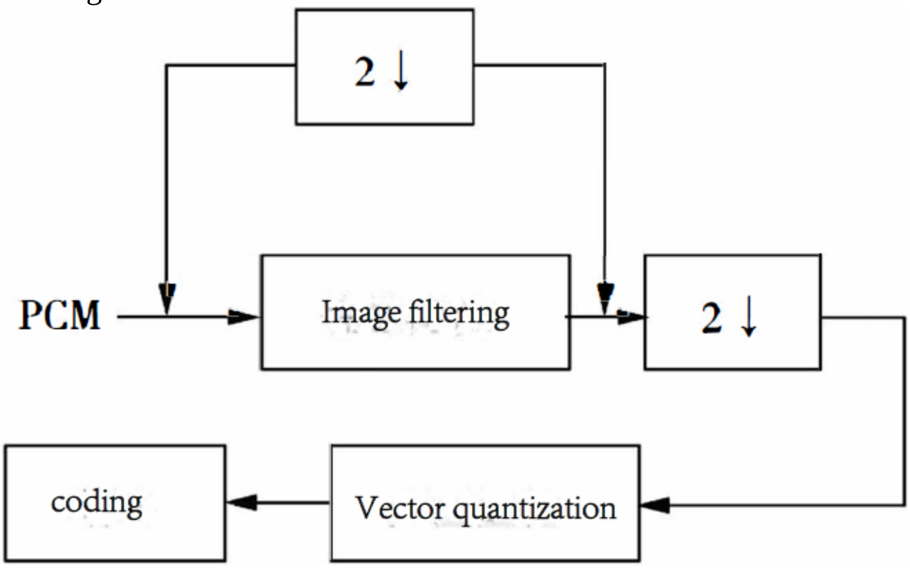

Fig.3 Wavelet Analysis Coding Structure Diagram

The symbol 2 above is expressed as wavelet coefficient decomposition. Through the above process, wavelet analysis coding can realize a smooth and detailed signal description form. Then the smooth signal is further decomposed, and a complete overlapping orthogonal transform process is obtained in the entire transform domain. From the perspective of vector, we can see that in K-space, there is a limited subset of mapping, so the vectorized mapping code can be obtained after encoding processing, and the complexity reduction can be achieved through a binary tree.

3.3 Application of Wavelet Analysis in Separating Different Components of Signal

Wavelet analysis is used to separate sinusoidal and noisy signals.

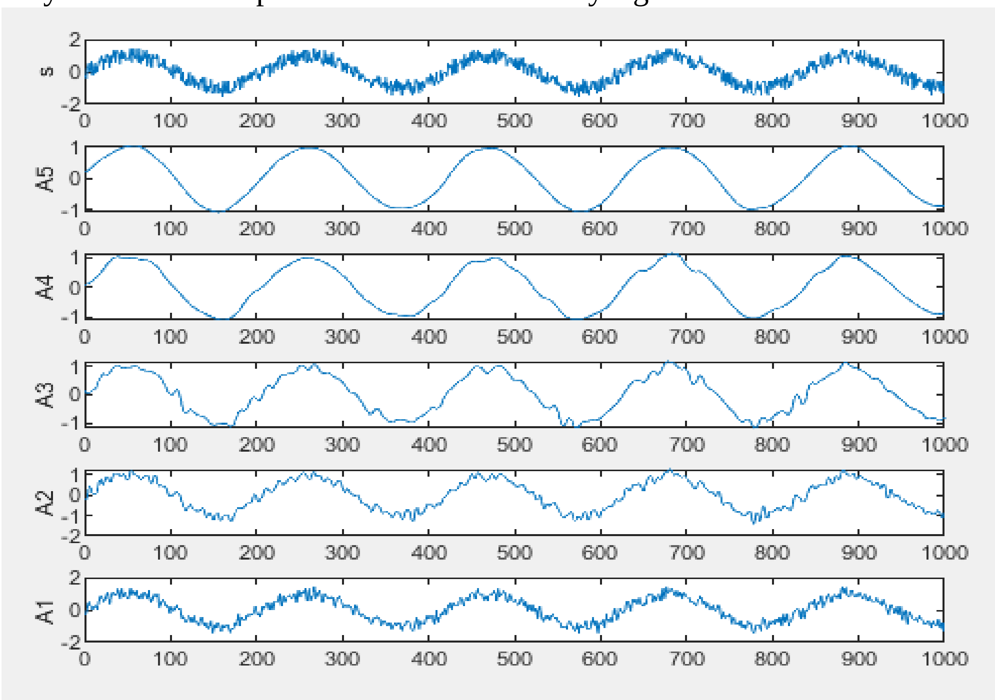

Fig.4 Decomposed Low-Frequency Coefficients

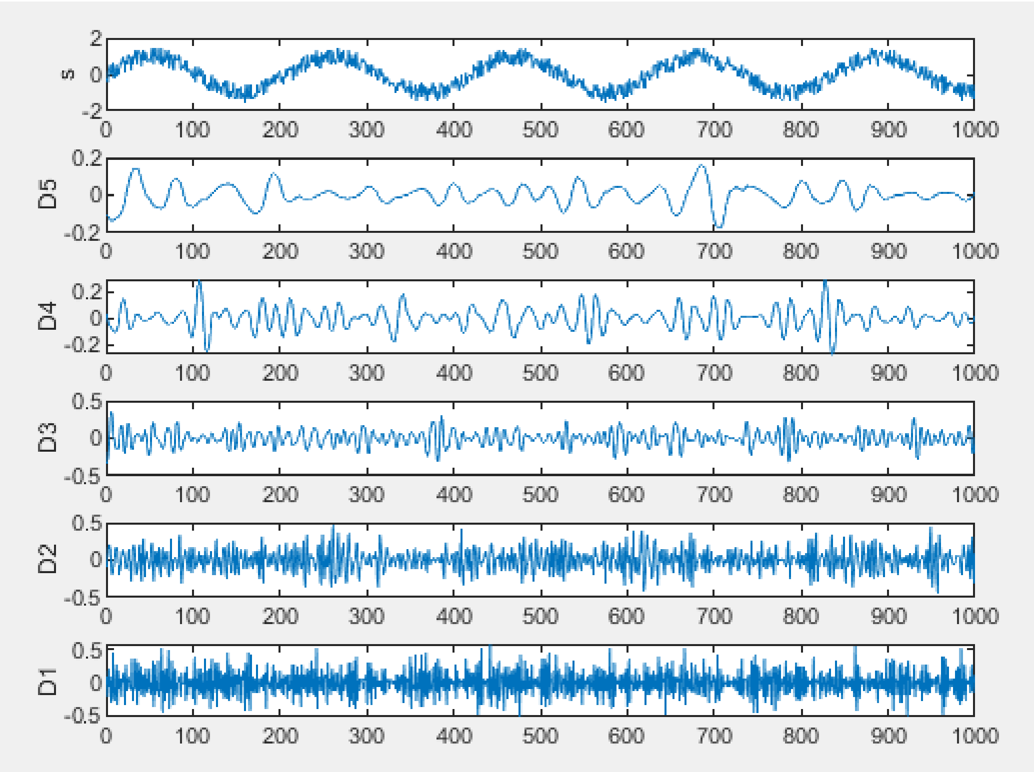

Fig.5 Decomposed High-Frequency Coefficients

Based on the idea of wavelet compression, a small number of approximation coefficients and some detail parameters are used to represent the original speech accurately. Therefore, wavelet decomposition plays a key role, and the decision of decomposition level depends on the type of speech signal and some other parameters. In this paper, db5 wavelet is used to decompose the speech signal in five levels, and then the low-frequency coefficients from the fifth level to the first level are reconstructed.

4. Conclusion

Through the simulation in Matlab, one-dimensional wavelet transform is obtained, which can denoise and compress the speech signal, and shows good parameter characteristics. While enhancing speech signal, in order to make the effect obvious, we especially add noise. In this process, only the wavelet generating function db3 is used to decompose. For speech signal, the five-level decomposition is enough, and the global threshold is used for signal enhancement. It is mainly realized by wavedec and wdencmp.

Wavelet analysis has both profound theory and wide application, and its theoretical research results and application range can't be accurately predicted, but it is certain that as a new excellent time-frequency analysis method, wavelet transform will continue to develop and improve, and make great contributions to the development of mathematics, signal processing and many other scientific fields. Compared with the application of wavelet transform in image processing, its potential in speech signal processing is infinite and its future is unlimited.

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