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Analytic Study of Waves in a Magnetised Partially Ionised Gas

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Abstract: Expanding Wave interaction in a magnetised partially ionised gas is simulated by introducing an oscillatory disturbance at a boundary surface. The basic Magneto hydrodynamic (MHD) equations are solved under realistic simple assumptions. The resulting dispersion expressions are then analysed for wave propagation phenomena in the plasma environment. The contributions of magnetic field, ionic-neutral mixes and collision frequencies are highlighted qualitatively. The outcome of the observations could find usefulness in communication, space vehicle re-entry and technology in general, with attendance enrichment in the literature in wave-wave interaction in the physics of astrophysical sites.

1. Introduction

It is now known that a small fraction of interstellar gas exists in the form of luminous, ionized and tenuous gas known as H_{11} region and the rest is mostly neutral hydrogen. Thus the Interstellar gas is not completely ionized and it is permeated with neutral atoms. A binary mixture of this situation in partially ionized gas calls for its idealization as a mixture of a hydro-magnetic component (charged) and a neutral component, the two interacting only through mutual collisions.
A magnetic field applied to this binary mixture interacts only with the charged particles and it is the collision of the ions with the predominantly neutral gas that is responsible for indirect coupling of the magnetic field with the bulk of the gas.

The problems of studying the complex occurrence of neutral and ionized species of those particles accelerated at astrophysical sources and those particles produced in interaction and collisions with those on interstellar gas are relevant to telecommunication, astrophysical observations, space vehicle re-entry communication, geodetic surveys and many more.

The behaviour of wave's interaction in such media is a phenomenon of great interest to astrophysicists, space scientists and theoretical physicists alike.

Very good literature is available on the issue of waves in partially ionized plasma (Reberto et al, 2013, Alagoa et al 1993., Alabraba etal, 2008, Zhi-Bin etal, 2016,). The effects of ionization, collision and magnetic fields etc, have been adduced in these references. The applications and consequences of these effects in technology and communication have been highlighted copiously.

The approach presented in this work, though adopts the Magnetohydrodynamic (MHD) model adds to the richness of the literature and gives a more general outlook, where the non-dimensional parameters could take care of a wide range of applications. For instance, the frequencies of the probing agent could cover the whole electromagnetic spectrum and even mechanical waves which is applicable to a wide range of situations in technology and communication, so also with β (representing various ion- neutral mixes).

Previous investigations show that frequency ranges of disturbance sources determine the modes and wave types propagated in plasma. (Alagoa et al, 1995, Skukla et al 1996, Zhi-Bin et al, 2016).

Penetration of waves in a plasma are seen to be governed also by viscosity, conductivity for fully ionised plasma, but collision effects and ionisation levels get more pronounced in partially ionised plasma.

Pekene and Ekpe (2015) investigated unsteady state flow of radiating partially-ionized plasma in the galactic centre in the presence of inclined magnetic field and considered the influence of collision, frequency and radiative effects.

The effect of collision on the onset of stationary cells diminishes for optical thin non-grey plasma-near Steady state. This is of relevance and importance in cosmic ray physics as the interaction between the ionized and neutral gas components represents a state which often exists in the universe.

MHD equations and fluid equations are generally non-linear, this means that the solutions are complex and also difficult to evaluate.

First, we give the mathematical formulation of the problem, attempt a method of solution and give analysis of the results.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

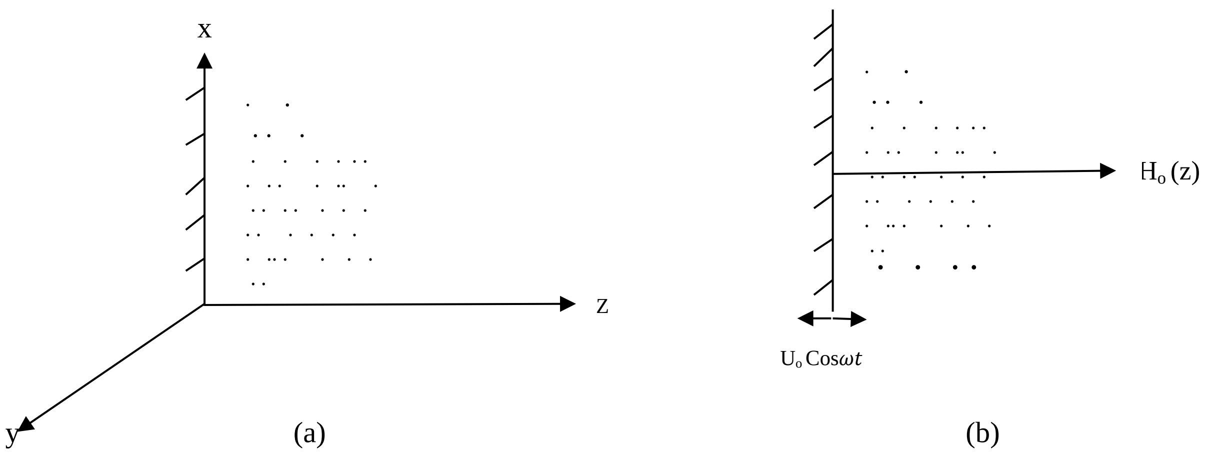

Figure (1a, b): Physical model

We consider (in form of a simulation model), a two dimensional Cartesian coordinate flow of binary plasma which is under the influence of homogeneous magnetic field. The field (H_0) is applied normal to the plane bounding the plasma. Waves are excited at the bounding surface which executes harmonic oscillations of the frequency ω along its own plane described by $(u_0 \cos\omega t)$, where u_0 is the amplitude of vibration.

An external magnetic field H_0 is applied along the axial direction with induction H_x and H_y in directions perpendicular (orthogonal) to the applied field. The dimensionless (unrestricted) equation governing the flow, considering incompressible and viscous gas is as given by Alagoa et al (1993, 1995).

$$\frac{\partial^2 Q_i}{\partial z^2} + N^2\bar{\upsilon}\frac{\partial p}{\partial z} - \sigma_n^2\bar{\upsilon}\frac{\partial Q_i}{\partial t} + \sigma_i^2\bar{\upsilon}^2\beta(Q_n - Q_i) = 0 \; \ldots .1$$

$$\frac{\partial^2 Q_n}{\partial z^2} - \sigma_n^2\frac{\partial Q_n}{\partial t} - \sigma_i^2\bar{\upsilon}(Q_n - Q_i) = 0 \; \ldots\ldots\ldots\ldots.\ldots 2$$

$$(\xi - ik|\upsilon_i)\frac{\partial^2 p}{\partial z^2} + \bar{\upsilon}\frac{\partial Q_i}{\partial z^2} - \sigma_n^2\bar{\upsilon}\frac{\partial p}{\partial t} = 0 \; \ldots\ldots\ldots\ldots\ldots 3$$

Here $Q_{i,n} = u_{i,n} + jv_{i,n}$, $P = H_x + jH_y$

where, $Q_{i,n}$ represent the two dimensional velocities u, v of the species, i and n $-$ represent respectively, ionised and neutral components, P is induced magnetic fields along x and y directions.

$A_0 = H_0(\mu_m|\rho_i)^{\frac{1}{2}}$

$\sigma_{i,n}^2 = \omega_{i,n}\upsilon_{i,n}|u_0^2$, a wormesly type parameter.

$k = H_0|\rho_i$ a Hall parameter.

$\upsilon = \mu_{i,n}|\rho_{i,n}$ is kinematic viscosity.

μ_m is a magnetic permeability.

$\mu_{i,n}$ is viscosity of the species

$\xi = \eta|\upsilon_i$,$\upsilon_{i,n}|\upsilon = \bar{\upsilon}_{i,n}$, $\beta = \frac{\rho_n}{\rho_i}$ is a ratio of species density indicative of degree of ionization of the gas.

The equations (1-3) are subject to the conditions

$Q_{i,n} = Cos\ t = \frac{1}{2}\{e^{it} + e^{-it}\}$ on $z = 0$

$P = 0$,on $z = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .4$

3. METHOD OF SOLUTION

First, we separate the space dependent quantities from the time dependent quantities by supposing.

$$Q_{i,n} = \tfrac{1}{2}\{q_{i,n}(z)e^{jt} + \bar{q}_{i,n}(z)e^{-jt}\}\ldots\ldots.5$$

$$P = \tfrac{1}{2}\{P(z)e^{jt} + \bar{P}\,(z)e^{-jt}\}\ldots\ldots\ldots\ldots.6$$

and substitute in 1-3 to obtain.

$$\left(\frac{d^2}{dz^2} - j\sigma_n^2\bar{\upsilon} - \sigma_i^2\bar{\upsilon}^2\beta\right) q_i(z) + N^2\bar{\upsilon}\frac{dp_{(z)}}{dz} + \sigma_i^2\bar{\upsilon}^2\beta q_n(z) = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.7$$

$$\left(\frac{d^2}{dz^2} - j\sigma_n^2 - \sigma_i^2\bar{\upsilon}\right) q_n\,(z) + \sigma_n^2\bar{\upsilon}q_i = 0\ldots.8$$

$$\left(\lambda\frac{d^2}{dz^2} - j\sigma_n^2\right)p_{(z)} + \bar{\upsilon}\frac{dq_i(z)}{dz} = 0 \;\ldots\ldots\ldots.9$$

where $\lambda = \xi - jk|\upsilon_i$

Solving 7-9 simultaneously results in

$$[\lambda \frac{d^6}{dz^6} - (\lambda j\sigma_n^2\bar{\upsilon} + \lambda\sigma_i^2\bar{\upsilon}^2\beta + \lambda j\sigma_n^2 + \lambda\sigma_i^2\bar{\upsilon} + \bar{\upsilon}^2\mathrm{N}^2 + j\sigma_n^2\bar{\upsilon})\frac{d^4}{dz^4} + \{(j\sigma_n^2 + \sigma_i^2\bar{\upsilon})(j\sigma_n^2\bar{\upsilon} + \sigma_i^2\bar{\upsilon}^2\beta)\lambda - \sigma_i^4\bar{\upsilon}^3\beta\lambda - \sigma_n^4\bar{\upsilon}^2 + j\sigma_i^2\sigma_n^2\bar{\upsilon}^3\beta - \sigma_n^4\bar{\upsilon} + j\sigma_i^2\bar{\upsilon}\sigma_n^2 + j\sigma_n^2\bar{\upsilon}^2\mathrm{N}^2 + \sigma_i^2\bar{\upsilon}^3\mathrm{N}^2\}\frac{d^2}{dz^2} - \{(j\sigma_n^2 + \bar{\upsilon}\sigma_i^2)(j\sigma_n^2 + \sigma_i^2\bar{\upsilon}^2\beta) + j\bar{\upsilon}\sigma_n^2 + j\sigma_i^4\bar{\upsilon}^4\beta\sigma_n^2\}]P(z) = 0 \text{ …………………10}$$

Equation (10) is a bi-cubic equation with complex coefficients. The six order differential equation can however be reduce to a forth order if it is assumed that λ is negligible, which imposes the condition that the Hall current is very small. Therefore, letting $\lambda \to 0$ the equation 10 reduces to a bi-quadratic equation whose characteristic roots are given by;

$$m_{1,2,3,4} = \pm\{-\frac{\alpha_2}{2\alpha_1} \pm \sqrt{\frac{\alpha_2^2 + 4\alpha_1\alpha_3}{4\alpha_1^2}}\}^{\frac{1}{2}} \text{ ……11}$$

where $\alpha_1 = \bar{\upsilon}\mathrm{N}^2 + j\sigma_n^2\bar{\upsilon}$

$\alpha_2 = (\bar{\upsilon}\sigma_n^2 + \bar{\upsilon}\sigma_n^4 + \bar{\upsilon}^3\sigma_i^2\mathrm{N}^2) - j(\bar{\upsilon}^3\sigma_n^2\sigma_i^2\beta + \bar{\upsilon}^2\sigma_n^2\sigma_i^2 + \bar{\upsilon}^2\sigma_n^2\mathrm{N}^2)$.

$\alpha_3 = (\sigma_n^4\sigma_i^2\bar{\upsilon}^3 + \sigma_n^4\sigma_i^2\bar{\upsilon}^3\beta) - j(\bar{\upsilon}^3\sigma_n^2\sigma_i^4\beta - \bar{\upsilon}^4\sigma_n^2\sigma_i^4\beta - \sigma_n^6\bar{\upsilon}^2)$.

The magnetic field is finite at large distances from the boundary $z = 0$, hence the only appropriate roots can be written as.

$$P(z) = Aexp\{-m_1 z\} + Bexp(-m_2 z)\text{…12}$$

If equation (12) is now substituted in (7-9) and applying conditions in equation (4) result finally in;

$$P(z) = \frac{j\sigma_n^2}{\mathrm{N}^2\bar{\upsilon}(m_2 - m_1)}\{\exp(-m_1 z) - \exp(-m_2 z)\} \text{ … … … ..13}$$

$$q_i(z) = \frac{\sigma_n^4}{\mathrm{N}^2\bar{\upsilon}^2(m_2 - m_1)}\{\frac{1}{m_1}\exp(-m_1 z) - \frac{1}{m_2}\exp(-m_2 z) + \frac{m_1 - m_2}{m_1 m_2}\} + 1 \text{…………..14}$$

$$q_{n(}z) = \left(1 + \frac{j\sigma_n^2}{\sigma_i^2\bar{\upsilon}\beta}\right)\left\{\frac{\sigma_n^4}{\bar{\upsilon}^2\mathrm{N}^2}\left(\frac{e^{-m_1 z}}{m_1} - \frac{e^{-m_2 (z)}}{m_2} + \frac{m_1 - m_2}{m_1 m_2}\right) + 1\right\} - \frac{j\sigma_n^2}{\sigma_i^2\beta\bar{\upsilon}(m_2 - m_1)}(m_2 \mathrm{e}^{-\mathrm{m}_2 \mathrm{z}} - m_1 e^{-m_1 z}))\text{…..15}$$

If we further express.

$m_{1,2} = a_{1,2} + jb_{1,2}$ … … … … … … … … ….16

Then

$P(z)e^{jt} = A_1\{e^{-a_1 z}e^{-j(b_1 z - t)} - e^{-a_2 z}e^{-j(b_2 z - t)}\}$………………………....17

Similarly

$q_i(z)e^{it} = B_1 e^{-a_1 z}e^{-j(b_1 z - t)} - B_2 e^{-a_1 z}e^{-j(b_2 z - t)} + B_3 e^{jt}$…………….18

$q_n(z)e^{jt} = C_1 e^{-a_1 z}e^{-j(b_1 z - t)} - C_2 e^{-a_2 z}e^{-j(b_2 z - t)} + C_3 e^{jt}$……………..19

(See the expressions for the coefficients in the appendix)

To obtain final solutions for $Q_{i,n}$ and P as in equation 4 the complex conjugate pairs of the expressions $(17-19)$ must be obtained.

What is however clear is the appearance of decay oscillations with distance z. Pure sinusoids also exist in the velocity.

The amplitude of the various modes of oscillations varies with z and depends entirely on the values of a_1 and a_2. The velocity amplitudes of each wave mode differ from one another along the z direction.The phase velocity of each normal decay mode and its corresponding decay lengths can however, be computed from information in $b_{1,2}$ and $a_{1,2}$ respectively. Therefore, expanding equation (11) and retaining appropriate terms result in the following approximations.

$$m_1 = \sqrt{\alpha_3}, m_2 = j\sqrt{\frac{\alpha_2}{\alpha_1} + \alpha_3} \text{ ....20}$$

$$\text{With} |2\alpha_1^2\alpha_3^2| \gg |\alpha_2^2| \text{....................21}$$

If we allow $\upsilon_n = \upsilon_i = \upsilon$ and setting $\omega\upsilon = U_0^2$ (on dimensional consistency) the following are obtained;

$$a_1, b_1 = [\tfrac{1}{2}\{(1+\beta)^2(\frac{\omega_i^2}{\omega^2})+1 - 4\beta\frac{\omega_i^2}{\omega^2} + 4\beta^2\frac{\omega_i^4}{\omega^4}\}\pm\tfrac{1}{2}(1+\beta)\frac{\omega_i}{\omega^2}] \text{..................22}$$

$$a_2, b_2 = \{[\tfrac{1}{2}\{(1+\beta)^2\frac{\omega_i^2}{\omega^2} + (1-2\beta\frac{\omega_i^2}{\omega^2})^2 + \frac{2}{N^2+1}[\left(1-2\beta\frac{\omega_i}{\omega}\right)(1+\beta)N^2\frac{\omega_i}{\omega} - 2 + N^2(N^2-\frac{\omega_i}{\omega}) - (1+\beta)\frac{\omega_i}{\omega}(N^2+\frac{\omega_i}{\omega}.\left(N^4-\beta-1\right))] + \frac{1}{(N^4+1)^2}\{N^2+\frac{\omega_i}{\omega}(N^4-\beta-1)\}^2 + \{(1+\beta)N^2\frac{\omega_i}{\omega} - 2 + N^2(N^2-\frac{\omega_i}{\omega})\}^2\}]^{\frac{1}{2}} \pm \tfrac{1}{2}(1+\beta)\frac{\omega_i}{\omega} - \frac{1}{2(N^4+1}(N^4+\frac{\omega_i}{\omega}(N^4-\beta-1))]^{\frac{1}{2}} \text{.......................23}$$

Here it needs to be reminded that ω_i is the collision frequency between charged and neutral gas (with neutral as the reference frame), while ω is the forcing frequency of the external agent.

Equation (22-23) predict that, the phase velocities and decay or penetration lengths of the resulting modes of waves generated in the plasma (partially ionized gas) are dependent on the ratios of the collisions between species, the external frequencies of the forcing agent and the degree of gas ionization expressed in β and, $N = A_0|\omega\upsilon($ ratio between Alfven speed and characteristics speed defined by the forcing agent, otherwise refered to as the magnetic mach number).

4. ANALYSIS OF RESULTS

4.1 Wave propagated at frequencies $\omega \gg \omega_i$

If we take the case when $\omega_i|\omega \gg 1$, corresponding to high frequency wave generated in the plasma environment the following results are obtained;

$$a_1, b_1 = \sqrt{\tfrac{1}{2}}$$

$$a_2, b_2 = [\tfrac{1}{2}(\frac{N^2}{N^2+1)^2} + 1 + \frac{2N^2-4}{(N^2+1)} + \frac{N^4}{(N^4+1)^4} - \frac{4N^2}{(N^4+1)^2}]^{\frac{1}{2}} \pm \frac{N^2}{2(N^4+1)}]^{\frac{1}{2}}$$

In this case, it appears evidently that wave modes are purely shear and modified Alfven waves. In particular, when $N = 0$ corresponding to the absence of applied magnetic field ($U_0 \gg A_0$), the two modes reduce to one shear wave mode.

For a wave generated at $\sqrt{\omega \upsilon} = A_0$, which are hydro-magnetic requires that N = 1; and therefore $a_2 = 0.780$ and $b_2 = 0.322$.

4.2 Wave at $\omega \lesssim \omega_i$.

If we let $\omega = \omega_i$ and substitute in (22) and (23) results in this case

$$a_1, b_1 = \left[\frac{1}{2}(2 - 2\beta + 5\beta)^{1/2} \pm (1 + \beta) \quad \right]^{1/2}$$

$$a_2, b_2 = [\frac{1}{2}\left(2 - 2\beta + 5\beta^2 N \right.$$
$$+ \frac{2N^2}{N^4 + 1}\left(N^4 - N^2 + \beta - \beta^2 - (1 + \beta)(N^4 + 2N^2) \right.$$
$$\left.\left. + \frac{1}{(N^{4+1})^2}\{(N^4 + N^2 - \beta - 1)^2 + (N^2(1 + \beta) - 2 + N^4 - N^2)\}\right)\right)^{\frac{1}{2}} \pm \frac{1}{2}((1 + \beta)$$
$$- \frac{1}{2N^4 + 2}\{2N^4 - \beta - 1\})]^{1/2}$$

Here, we observe two wave modes, one of which is purely shear whose velocity and damping length depends on β and the other mode is hydro-magnetic with phase speed depending on β and magnetic Mach Number N.

If we choose β =1 for particular interest and let N = (0, 1) the following are obtained

$$a_1, b_1 = 1.45, 0.32$$
$$a_2, b_2 = 1.78, 0.32$$

and

$$a_1, b_1 = 1.45, 0.32$$
$$a_2, b_2 = 0.50(2 + j1.33)^{1/2}, 0.50(j1.33 - 2)^{1/2}$$

Consequently, a wave generated at frequency equal to the plasma collision frequencies results in streaming, i.e. space dependent oscillations. The decay lengths and phase velocities vary conspicuously according to the frequencies, ratios of neutral and ion densities and magnetic fields, either induced or external.

5. GENERAL OBSERVATIONS AND COMMENTS

The spectral forms presented here (in a and b) can be utilized to determine plasma concentration in various astrophysical sites. Shock waves generation in re-entry vehicles do not ionize the gases completely we suppose at the shock fronts, and therefore affect communication.

The spectral analysis of the resulting waves presented here could reveal good information about the plasma sites such as; density, ionisation levels and magnetic fields frozen in the modes.

Wavelets analysis where transient conditions are considered could hold very good promise in astrophysical plasma sites investigation. In the literature, waves generation are either via pressure variation, temperature variation and magnetic forces.

The forcing agent presented in our work can take the analogous form of many varied forcing agents, be it of pressure variations, temperature gradients and magnetic forces in so far as the frequency information can be recovered.

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APPENDIX

$$A_1 = \frac{j\sigma_n^2}{N^2\upsilon[(a_2-a_1)+j(b_1-b_2)]}$$

$$B_1 = \frac{\sigma_n^4}{N^2\upsilon^2(a_1+jb_1)[(a_2-a_1)+j(b_1-b_2)]}$$

$$B_2 = \frac{\sigma_n^4}{N^2\upsilon^2(a_2+jb_2)[(a_2-a_1)+j(b_2-b_1)]}$$

$$B_3 = \left(\frac{\sigma_n^4[(a_1-a_2)+j(b_1-b_2)]}{N^2\upsilon^1(a_1+jb_1)(a_2+jb_2)[(a_2-a_1)+j(b_2-b_1)]}+1\right)$$

$$C_1 = \left(1+\frac{j\sigma_n^2}{\upsilon\beta\sigma_i^2}\right)\left[\frac{\sigma_n^4}{N^2\upsilon^1(a_1+ja_2)[(a_2-a_1)+j(b_2-b_1)]}\right]+\left[\frac{j\sigma_n^2(a_1+ja_2)}{\sigma_n{}^2\beta\upsilon[(a_2-a_1)+j(b_1-b_2)]}\right]$$

$$C_2 = \left(1+\frac{j\sigma_n^2}{\upsilon\beta\sigma_i^2}\right)\left[\frac{j\sigma_n^2(a_2+jb_2)}{\sigma_i{}^2\beta\upsilon[(a_2-a_1)+j(b_2-b_1)]}-\frac{\sigma_n^4}{N^2\upsilon\ \ [(a_2-a_1)+j(b_2-b_1)]}\right]$$

$$C_3 = \left(1+\frac{j\sigma_n^2}{\upsilon\beta\sigma_i^2}\right)\left[\left(\frac{(a_1-a_2)+j(b_1-b_2)}{[(a_1+jb_1)(a_2+jb_2)]}\right)\frac{\sigma_n^4}{N^2\upsilon\ \ [(a_2-a_1)+j(b_2-b_1)]}\right]$$