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Investigation of Left Ventricular Hypertrophy using Mean Deviation Function

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Abstract: In this paper we have projected a new methodology to characterize non-differentiable graphs. The continuous functions but non-differentiable at some points or every points may be characterized via mean deviation function. The mean deviation function is the deviation of the mean function from the original function. It is differentiable and approximates the given non-differentiable function. This is a new type of characterization of unreachable graphs. Since ECG graphs are continuous everywhere but non-differentiable at a few number of points thus in this paper we have used this new methodology to find some distinguishable measurements of left ventricular hypertrophy by comparing normal and problematic ECGs.

1. Introduction

The theory of characterization of continuous but non-differentiable functions was studied long year back by several mathematicians like Euler, Liouville and others[1-2]. All continuous functions may not be differentiable. They cannot be characterized by the integer order classical calculus at the non-differentiable points and they cannot also be represented graphically. Such type of problem is still an unsolved problem as these problems don't have any real interpretations. However the continuous but non-differentiable functions can be characterized by fractal dimension which are already shown by Feynman [9]. F. Ben Adda [3] has demonstrated mathematical model for fractal manifold to study such nowhere differentiable functions. However the problems with non-smooth curves cannot be solved with proper geometrical interpretation. So it’s characterization is an emerging problem. In this paper we have taken such problems by approximating a non-differentiable function by a differentiable function.

In real field, non-differentiability arises in many bio-medical signals like ECG, EEG etc., Brownian motion, earth quake pattern, crack pattern etc.[4-8]. To investigate those types of non-differentiable functions our main theme of this paper is to approximate them by a differentiable function, named as mean function. It has been described briefly in this paper. Here we have characterized ECG graphs by mean deviation function which is the deviation of the mean function from the original function and is elaborately described in this paper. As the Electrocardiograph is a non-differentiable graph, we can approximate this by the mean function.

The standard Electrocardiogram (ECG) signal structure is shown in the figure 1 below:

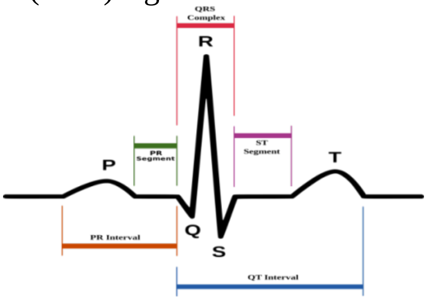


Figure 1: A normal shape of PQRST wave in ECG [10-11]

Electrocardiography is the process of recording the electrical activity of the heart over a period of time using electrodes placed on the skin. The PQRST wave structure in ECG graph gives several information of human hearts whether the patient may have any cardiac problem or not. The unreachable points of the ECG graphs are Q, R, S in the 'PQRST' wave. Classical integer order calculus is not applicable there because of non-differentiability properties at those points. In this paper we have been taken Q,R,S as points of the QRS complex of each ECG though they are waves of ECG. Here, we have formulated some propositions in terms of mean deviation function to observe some distinguishable character of normal ECG and Left Ventricular Hypertrophy. By calculating mean deviation functions at all non-differentiable points of each ECG under consideration we have tried to get some exquisite difference between them. The theory of mean function is described as follows:

1.1. Mean Function

Suppose we have an unreachable function[6,8] $y = f(x)$ on $[a,b]$. It cannot be interpreted by classical integer order calculus since slope of such type of function $\frac{df}{dx}$ cannot be determined at the non-differentiable points. But it is possible that these functions can be approximated by a differentiable function $f(x,\delta)$ where $x \in [a,b]$ & $\delta \in [0,1]$. This differentiable function is known as mean function. So we can always replace a non-differentiable function $f(x)$ by a differentiable mean function [3] given by, $f(x,\delta) = \frac{1}{2\delta}\int_{x-\delta}^{x+\delta} f(t)\,dt \qquad \forall x \in [a,b]\,;\delta \in [0,1]$

For $\delta \to 0$ we shall get back the unreachable function f(x), i.e. $\lim_{\delta \to 0} f(x,\delta) = f(x)$

Now we shall define forward and backward mean functions f^{+} and f^{-} as follows $f^{+}(x,\delta) = \frac{1}{\delta}\int_{x}^{x+\delta} f(t)dt$ and $f^{-}(x,\delta) = \frac{1}{\delta}\int_{x-\delta}^{x} f(t)dt$.

1.2. Mean Deviation

With the knowledge of approximating the unreachable function by the differentiable mean function, the mean deviation function[3] denoted by $\Delta_{\delta} f(t)$ can be defined as follows:

$$\Delta_\delta f(t) = f(t) - f^+(t,\delta)$$

With the above defined mean function and mean deviation function we have characterized unreachable functions in this paper considering both normal and problematic ECGs.

1.3. LVH ECG

In case of hypertrophy the left heart muscle becomes thicker. This type of hypertrophy can happen due to different causes such as an increasing workload in left ventricular, during hypertension or aortic valve stenosis. These causes are fundamentally different from hypertrophic obstructive cardiomyopathy (HCM), which is a congenital misalignment of cardiomyocytes, resulting in hypertrophy. The criteria for LVH ECG as follows [11]: i. Tall R wave (>25mm) in V5 or V6 lead, ii. Deep S wave in V1 or V2 lead, iii. Inverted T wave in leads I, II, AVL, V5, V6, sometimes in V4, iv. R waves in lead V5 or V6 plus S wave in lead V1 or V2 greater than 35 mm.

The organization of this paper is as follows: Some propositions are replaced in the section 2, section 3 is dedicated to the application of mean function to characterize the ECGs using tables and bar diagrams. Finally our paper has been concluded in section 4.

2. Proposition

Using the definition of mean function and mean deviation function we have developed some propositions to characterize unreachable ECG graphs. Both sides of each non-differentiable point of ECG graphs can be approximated either by a linear or a non-linear function. The function which is linear in both side of the unreachable point we have considered in type 1.

Type 1: Here we consider a function which is linear in both sides i.e. $f(x) = \begin{cases} ax+b, & p \le x \le q \\ cx+d, & q \le x \le r \end{cases}$

with $a \ne c$. It is continuous at $x = q$ if $aq + b = cq + d$ but not differentiable at that point. Then the mean function is; $f(x,\delta) = \frac{1}{2\delta}\left[\frac{c-a}{2}\left(x^2+\delta^2-q^2\right)+\delta x\left(c+a\right)+\left(x-q\right)\left(d-b\right)+\delta(b+d)\right]$

The forward mean function is given by; $f^+(x,\delta) = \frac{1}{\delta}\left[\frac{c-a}{2}\left(x^2-q^2\right)+\frac{c\delta^2}{2}+c\delta x+\left(x-q\right)\left(d-b\right)+d\delta\right]$

The mean deviation at the non-differentiable point x=q is given by $\Delta_\delta f(q) = -\frac{c\delta}{2}$.

This is the deviation of the forward mean function from the original function at x=q

Here we have considered only one type of functions. In ECG graphs another three types of functions are possible which are one side is linear and another is nonlinear of the unreachable points and both sides are non-linear of the unreachable points. The solutions of these above defined propositions have been used to interpret the LVH ECG and compare with the normal ECG. In each ECG we have found the mean deviation at each non-differentiable point which are shown in the next section.

3. Applications of mean deviation function in ECG Graphs

In this section we shall study non-differentiability nature of ECG graphs by mean function. Here we consider two type of ECGs in which one is normal and another is problematic (in our case it is

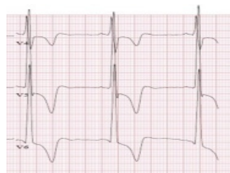
LVH ECG). We have constructed here a table of mean deviation at the non-differentiable points of each considerable leads using the above considerable propositions. Unreachable points Q,R,S of each PQRST wave of each lead are represented as points. The unreachable graphs i.e. normal and problematic ECGs are characterized by the table1,2 where table 1 has been constructed to show medical calculations and table 2 has been constructed to find the mean deviation at non-differentiable points Q, R, S of each PQRST wave of V1,V2,V5 and V6 leads. The second table is newly constructed by us from our given propositions. We have used the terminology 'NA' meaning Not Arise which is used when Q or S points are not observable clearly i.e. not prominent at QRS complexes of the ECGs.

Table 1: Length of waves of ECG: A=Lengths of S wave(mm) in V1 lead, B= Lengths of S wave(mm) in V2 lead, C=Lengths of R wave(mm) in V5 lead, D= Lengths of R wave(mm) in V6 lead


ECG sample I[12]

PQRST waves	A	B	C	D
1st	19.7	24.7	17.9	19.2
2nd	18.3	25.3	19.4	19.5
3rd	19.5	24.6	18.6	19.9

E=A+C	F=A+D	G=B+C	H=B+D
27.5	24	39.5	36
27.5	24	39.5	36
27.5	23	39.5	35
28.5	25	41	37.5
28.5	25	41	37.5
28.5	24	41	36.5

ECG sample II[12]

QRST waves	A	B	C	D
1st	12	27	56	56.5
2nd	11	25	54.1	56
3rd			59	53.2

E=A+C	F=A+D	G=B+C	H=B+D
68	68.5	83	83.5
66.1	68	81.1	83
71	65.2	86	80.2
67	67.5	81	81.5
65.1	67	79.1	81
70	64.2	84	78.2

Here, we see that in table-1 the maximum lengths of S wave in V1 and V2 of ECG sample I and II are respectively 19.7mm ;25.3mm and 12mm ;27mm. Also the maximum lengths of R wave in V5 and V6 of ECG sample I and II are respectively 19.4mm ;19.9mm and 59mm ;56.5mm. So these lengths are not very far from 25mm for sample I but it is very high for sample II compared to normal ECG. Moreover the maximum of the lengths of S1, S2, S3 and S4 for sample I and II are 45.2mm and 86mm respectively i.e. these lengths for Problematic sample ECG is very high. So the Doctor's point of view the ECG sample I is normal and the ECG sample II has cardiac problem which is Left Ventricular Hypertrophy.

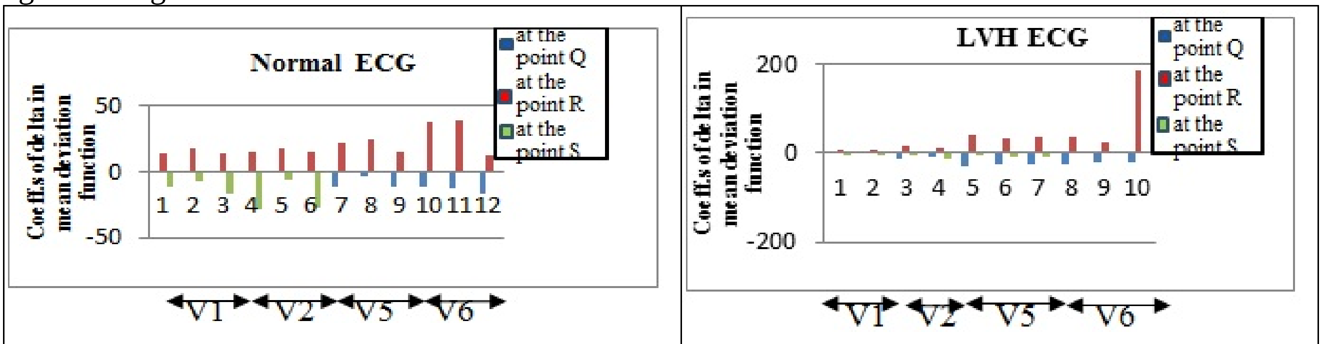
Now, we shall calculate the mean deviation function at the non-differentiable points of the normal ECG LVH ECG as given in table-2. In the tables-2 the values of mean deviation at the non-differentiable points are affected by δ . This δ is near to zero but not equal to zero. So the expressions along with this δ will be different for different values of δ . So we consider the coefficient of δ in those expressions of tables-2 to characterize ECG graphs.

Since we already know that LVH is characterized by tall R in V5 or V6 and deep S in V1 or V2, we have considered only those four leads in this paper to attain our conclusions and taken a look in the result of coefficient of δ in the mean deviation at the non-differentiable points i.e. S points in V1 or V2 leads and R points in V5 or V6 leads. Here we see that the V1 and V2 leads in the LVH ECG do not have any abnormal criteria which are already done above. Hence those coefficients of δ in the expressions of mean deviation at the non-differentiable points in V1 and V2 leads don't bring any significant result to distinguish LVH from normal ECG. If we consider those coefficients of δ in the expressions of mean deviation at the non-differentiable point R in V5 and V6 leads we see that the coefficient of δ in V5 and V6 leads are larger in LVH ECG than in normal ECG.

Table 2: Mean deviation function at the non-differentiable points of the normal LVH ECG

		Normal ECG (Sample I)				LVH ECG (Sample II)			
		V1	V2	V5	V6	V1	V2	V5	V6
1st	At Q point	NA	NA	-11.1875 δ	-12 δ	NA	-15 δ	-28 δ	-25.6818 δ
	At R point	14.0715 δ	15.4375 δ	22 δ	37.5 δ	6 δ	16.875 δ	38.9375 δ	36 δ
	At S point	-11.75 δ	-28 δ	-0.3572 δ	NA	-2 δ	-3.5715 δ	-3.25 δ	NA
2nd	At Q point	NA*	NA	-13.8572 δ	-12.1875 δ	NA	-7.5 δ	-27.25 δ	-23.3333 δ
	At R point	18.3 δ	18.0714 δ	24.25 δ	38.75 δ	7.1875 δ	11.3637 δ	34.3334 δ	25.5 δ
	At S point	-7 δ	-15.5 δ	0	NA	-1.8334 δ	-15 δ	-7.5 δ	NA
3rd	At Q point	NA	NA	-11.625 δ	-16.5833 δ			-26.8182 δ	-20.4615 δ
	At R point	13.9286 δ	15.375 δ	15.25 δ	13 δ			36.7778 δ	186 δ
	At S point	-17 δ	-27 δ	0	NA			-8.5 δ	NA

Now we have constructed bar-diagram of this coefficient of δ to get better conclusions which are given in figure 2.



(a) (b)

Figure 2: Coefficients of δ (delta) in mean deviation function at the non-differentiable points of the (a) normal ECG graphs and (b) Left Ventricular Hypertrophy ECG graphs

This bar-diagram shows clearly that coefficients of δ in mean deviation functions at the non-differentiable point R in V5 and V6 leads for normal ECGs is lying in a small range whereas this range is broader for the LVH ECG. Also we see from the bar diagram that for LVH ECG the maximum of the coefficient of δ in the expression of mean deviation function at the non-differentiable point R in V5 and V6 lead is 186 which is shown in table 2B. It is very high compared to normal ECG. Thus for a LVH patient coefficients of δ in the mean deviation function at non-differentiable point will be very high which in this case is 186 nearly 200 while for normal ECG it is below 40.

4. Conclusions

In this paper we have studied characteristics of normal ECG graph and LVH ECG graphs, by the help of mean deviation function. Here we see that in normal ECG the coefficient of δ in the expression of the mean deviation function at the non-differentiable point R of V5 and V6 leads are less than those for problematic ECG (in our case LVH). This results clearly show from the bar diagrams in the previous section. Hence it is possible that we can get a suitable range for this type of characterization of left ventricular hypertrophy ECG by examining large numbers of ECGs for helping the Doctors to determine LVH conditions of patients. We will report other cardiac ailments in our next study. This is a new type of study which is not reported till now by any one. It is a new method which we are reporting for the first time-could be an aid for differential diagnostics in medical science.

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Compliance with Ethical Standards

Authors declare that none of them have any conflict of interest.

Ethical approval

All ECG have been downloaded from internet

References

[1] S. Das(2011). Functional Fractional Calculus 2nd Edition, Springer-Verlag.

[2] S. Sengupta,U. Ghosh, S. Sarkar and S. Das(2018).A Mathematical Approach to Characterize Left Ventricular Hypertrophy from ECG Diagrams.Open Journal of Cardiology & Heart Diseases. Vol. 1; Issue No. 2.

[3] F. Ben Adda(2007). Mathematical Model for Fractal Manifold, International Journal of Pure and Applied Mathematics, vol.38 (2), 155-186.

[4] S. S. Ray, R.K. Bera(2005). An approximate solution of a nonlinear fractional differential equation by Adomian decomposition method. Applied Mathematics and Computation. 167 561–571.

[5] V. S. Erturk and S. Momani(2008).Solving systems of fractional differential equations using differential transform method. Journal of Computational and Applied Mathematics 215 142 – 151

[6] S. Sengupta, U. Ghosh, S. Sarkar and S. Das(2018), "Application of fractional calculus to distinguish left ventricular hypertrophy with normal ECG," 4th International Conference on Recent Advances in Information Technology (RAIT), Dhanbad, 2018, pp. 1-6.

[7] G. Jumarie(2006), Modified Riemann-Liouville derivative and fractional Taylor series of non-differentiable functions further results, Computers and Mathematics with Applications, 51. 1367-1376.

[8] U. Ghosh, S. Sengupta, S. Sarkar and S. Das(2015). Characterization of non-differentiable points of a function by Fractional derivative of Jumarie type. European Journal of Academic Essays. 2(2): 70-86.

[9] R.P. Feynman, Quantum Electrodynamics, W.A. Benjamin, New York (1961)

[10] B. B. Mandelbrot, (1982). The Geometry of Nature. Freeman. San Francisco

[11] J. R. Hampton (2010). The ECG made easy. Eighth ed.

[12] ECGs are available from Internet (free Google Images).