

Research on the Digital Hydraulic Cylinder Nonlinear Robust Controlling Methods with Input Saturation

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Abstract: First, the nonlinear model of the digital hydraulic cylinder and the controller are constructed, then the nonlinear problem is translated into the solving of a linear matrix inequality. Finally the anti-windup compensator is designed. Simulation results show that the proposed controller and compensator guarantee the robustness and good dynamic behavior of the system.

1. Introduction

With the development of the digital hydraulic technique, different methods are applied in the controlling of the digital hydraulic cylinder; however, most of them are based on the linear model with an absence on the certification of the system's nonlinear element and stability under the input saturation. The present essay is to design a nonlinear controller and an anti-windup compensator, which not only guarantee the system's stability, but also fulfill the system's dynamic requirements.

2. The Nonlinear Model of the Digital Hydraulic Cylinder with Uncertain Parameter

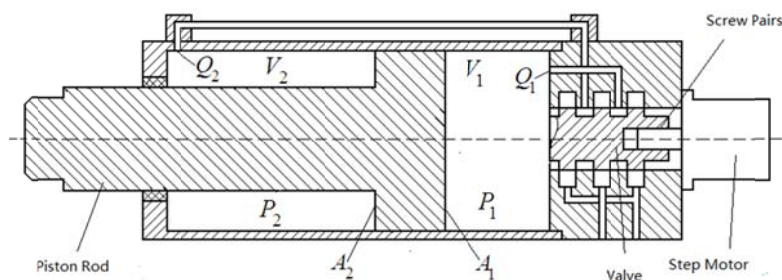


Fig 1 The inner structure of the digital hydraulic cylinder

The Fig. 1 is the inner structure of the digital hydraulic cylinder. P_1 and P_2 are the press of the non-rod chamber and the rod chamber, Q_1 and Q_2 are their flow, V_1 and V_2 are their volume, A_1 and A_2 are their effective area. The nonlinear model is as follows^[1].

$$\begin{cases} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{cases} = \begin{cases} x_2 \\ \frac{A}{M}(x_3 - \varepsilon x_4) - \frac{k_s}{M}x_1 - \frac{B_p}{M}x_2 - F_L/M \\ -\frac{\beta_e}{V_1}[Ax_2 + C_i(x_3 - x_4)] + u \frac{\beta_e R_1}{V_1} \begin{cases} \sqrt{P_s - x_3} & x_v \geq 0 \\ \sqrt{x_3 - P_0} & x_v < 0 \end{cases} \\ \frac{\beta_e}{V_2}[A_2x_2 + C_i(x_3 - x_4) - C_e x_4] - u \frac{\beta_e R_1}{V_2} \begin{cases} \sqrt{x_4 - P_0} & x_v \geq 0 \\ \sqrt{P_s - x_4} & x_v < 0 \end{cases} \end{cases} \quad (1)$$

In Eq.(1): x_1 is the displacement of rod; x_2 is the speed of rod; $x_3 = P_1$; $x_4 = P_2$. M is the equivalent quality; k_s is the spring stiffness; B_p is the damping coefficient; $\varepsilon = A_2/A_1$; β_e is the elastic modulus of oil volume; $V_1 = V_{01} + A_1x_1$, $V_2 = V_{02} + A_2x_1$; V_{01} and V_{02} are relatively the non-rod chamber and the rod chamber's initial volume; $R_1 = \frac{C_d w_f t_1}{\sqrt{2\rho\pi}}$, C_d is the flow coefficient of valve port, $w_f = \pi d_v$ is the valve core area gradient, d_v is the valve core diameter, ρ is the density of hydraulic oil, t_1 is the lead of valve core; P_s and P_0 are the oil pressure source and the Return pressure; u is the input rotation angle of step motion; F_L is the load force; C_i is the internal leakage coefficient; C_e is the external leakage coefficient. Let the input displacement signal be r , the tracking error signal be $e_1 = x_1 - r$, $e_2 = x_2 - \dot{r} = \dot{e}_1$, $e_3 = x_3 - \ddot{r} = \dot{e}_2$. Let Eq.(1) Substituted:

$$\begin{cases} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{cases} = \begin{cases} e_2 \\ e_3 \\ \tau_1 e_1 + \tau_2 e_2 + \tau_3 e_3 + A_1/M(g_3 - \varepsilon g_4)u + w \end{cases} \quad (2)$$

Among them:

$$\begin{cases} \tau_1 = -\beta_e C_i \frac{k_s}{M} \\ \tau_2 = -\left[\frac{k_s}{M} + \frac{\beta_e}{M} A_1 \left(\frac{A_1}{V_1} + \frac{\varepsilon A_2}{V_2} \right) + \frac{\beta_e B_p}{M} C_i \left(\frac{1}{V_1} + \frac{\varepsilon}{V_2} \right) \right] \\ \tau_3 = -\left[\frac{B_p}{M} + \beta_e C_i \left(\frac{1}{V_1} + \frac{\varepsilon}{V_2} \right) \right] \\ g_3 = \frac{\beta_e R_1}{V_1} \begin{cases} \sqrt{P_s - x_3} & x_v \geq 0 \\ \sqrt{x_3 - P_0} & x_v < 0 \end{cases} \\ g_4 = \frac{\beta_e R_1}{V_2} \begin{cases} \sqrt{x_4 - P_0} & x_v \geq 0 \\ \sqrt{P_s - x_4} & x_v < 0 \end{cases} \\ w = \frac{A_1}{M} \beta_e x_4 \left[x_4 C_i (1 - \varepsilon) \left(\frac{1}{V_1} + \frac{\varepsilon}{V_2} \right) + x_4 \frac{\varepsilon C_e}{V_2} - \left(\frac{A_1}{V_1} + \frac{\varepsilon A_2}{V_2} \right) \ddot{r} \right] - \frac{\beta_e C_i}{M} \left(\frac{1}{V_1} + \frac{\varepsilon}{V_2} \right) (k_r r + B_p \dot{r} + F_L + M\ddot{r}) \end{cases}$$

In Eq.(2), take w as disturbance, which is bounded. For the existence of input saturation, suppose u 's value range is:

$$|u| \leq u_{\max} \quad (3)$$

In the hydraulic system, some complex elements caused the nominal value[2] in Eq.(2). M, β_e, C_d, R_1 are the uncertain parameters considered in the present essay, they are expressed as:

$$\begin{aligned} \frac{1}{M} &= \frac{1}{\bar{M}}(1 + \Delta_1) \\ \beta_e &= \bar{\beta}_e(1 + \Delta_2) \\ C_d &= \bar{C}_d(1 + \Delta_3) \\ R_1 &= \bar{R}_1(1 + \Delta_4), \bar{R}_1 = \frac{\bar{C}_d w_f t_1}{\sqrt{2\rho\pi}} \end{aligned} \quad (4)$$

The nominal parameters are respectively $\bar{M}, \bar{\beta}_e, \bar{C}_d, \bar{R}_1$. $\Delta_1, \Delta_2, \Delta_3$ describe respectively the uncertainty set of M, β_e, C_d and $|\Delta_1| \leq a_1, |\Delta_2| \leq a_2, |\Delta_3| \leq a_3$. The certainty of $\tau_1, \tau_2, \tau_3, g_3$ and g_4 are $\bar{\tau}_1, \bar{\tau}_2, \bar{\tau}_3, \bar{g}_3, \bar{g}_4$, their uncertainty are $\Delta\tau_1, \Delta\tau_2, \Delta\tau_3, \Delta g_3, \Delta g_4$, with $|\Delta\tau_1| \leq \tau_{m1}, |\Delta\tau_2| \leq \tau_{m2}, |\Delta\tau_3| \leq \tau_{m3}$, Eq. (5) is as follows:

$$\begin{aligned} \Delta\tau_1 &= \tau_1 - \bar{\tau}_1, \Delta\tau_2 = \tau_2 - \bar{\tau}_2, \Delta\tau_3 = \tau_3 - \bar{\tau}_3 \\ \Delta g_3 &= g_3 - \bar{g}_3, \Delta g_4 = g_4 - \bar{g}_4 \end{aligned} \quad (5)$$

3. The Design of the Digital Hydraulic Cylinder' Nonlinear Tracking Controller

Eq. (2) could be constructed into linear term The constructed controller is as follows:

$$u = \frac{\bar{M}A \cdot v}{g_3 - \varepsilon \bar{g}_4} \quad (6)$$

In Eq. (6), v is the unknown and is to meet the constraint $v_{\min} \leq v \leq v_{\max}$, $v_{\max} = A/\bar{M}|\bar{g}_3 - \varepsilon\bar{g}_4|u_{\max}$, $v_{\min} = -A/\bar{M}|\bar{g}_3 - \varepsilon\bar{g}_4|u_{\max}$. Let Eq. (6) Substituted into Eq. (2) :

$$\begin{aligned} \dot{\hat{x}} &= (A + \Delta A)\hat{x} + (B_2 + \Delta B_2) \text{sat}(v) + \hat{B}_1 w \\ z &= C_2 \hat{x} \end{aligned} \quad (7)$$

Among them:

$$\hat{x} = [e_1, e_2, e_3]^T, A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \bar{\tau}_1 & \bar{\tau}_2 & \bar{\tau}_3 \end{bmatrix}, \Delta A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Delta\tau_1 & \Delta\tau_2 & \Delta\tau_3 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \Delta B_2 = \begin{bmatrix} 0 \\ 0 \\ (1+\Delta_1)(1+\Delta_2)(1+\Delta_3)-1 \end{bmatrix}, \text{sat}(v) = \begin{cases} v & |v| \leq v_{\max} \\ \text{sgn}(v)v_{\max} & |v| > v_{\max} \end{cases}, \hat{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C_2 = [1 \ 0 \ 0]$$

If v could stabilize Eq.(7), u could definitely stabilize Eq.(1). Therefore, the resolution is the solving to the parameter v in Eq.(7). The construct controller v is as follows:

$$\begin{cases} \dot{\eta} = A_c \eta + B_c \hat{x} + E_c (v - \text{sat}(v)) \\ v = C_c \eta + D_c \hat{x} \end{cases} \quad (8)$$

As in Eq. (8), A_c, B_c, C_c, D_c are the unsolved matrix in the controller, the static compensation controller is $E_c(v - \text{sat}(v))$, E_c is also the unsolved matrix. Firstly, the saturation of v is not considered to the design of the controller, A_c, B_c, C_c, D_c are obtained. In the next section, the compensator will be designed to the controller. Apply Eq.(8) to Eq.(7), Eq.(9) is obtained:

$$\begin{cases} \dot{\xi} = A_\xi \xi + B_{\xi w} w \\ z = C_\xi \xi \end{cases} \quad (9)$$

Among them:

$$\xi = \begin{bmatrix} \hat{x} \\ \eta \end{bmatrix}, A_\xi = \begin{bmatrix} (A + \Delta A) + (B_2 + \Delta B_2) D_c & (B_2 + \Delta B_2) C_c \\ B_c & A_c \end{bmatrix}, B_{\xi w} = \begin{bmatrix} \hat{B}_1 \\ 0 \end{bmatrix}, C_\xi = [C_2 \ 0], \bar{A}_\xi = \begin{bmatrix} A + B_2 D_c & B_2 C_c \\ B_c & A_c \end{bmatrix} = \bar{A}_{\xi 0} + F_1 K_c F_2, \Delta A_\xi = \begin{bmatrix} \Delta A + \Delta B_2 D_c & \Delta B_2 C_c \\ 0 & 0 \end{bmatrix} = F_3 F_4 + F_5 K_c F_2,$$

$$\bar{A}_{\xi 0} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}_{6 \times 6}, K_c = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}_{4 \times 6}, F_1 = \begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix}_{6 \times 4}, F_2 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}_{6 \times 6}, F_3 = [0, 0, 1, 0, 0, 0]^T, F_4 = [\Delta\tau_1, \Delta\tau_2, \Delta\tau_3, 0, 0, 0], F_5 = \begin{bmatrix} 0 & \Delta B_2 \\ 0 & 0 \end{bmatrix}_{6 \times 4}$$

The necessary and sufficient condition which ensures that Eq.(5) is stabled and L_2 gain is smaller than γ is ^[3-4]: there exists Lyapunov function $V(\xi) > 0, \dot{V}(\xi) < 0$. Let $V(\xi) = \xi^T P \xi$, P is the positive definite symmetric matrix. If $\dot{V}(\xi) < 0$ is tenable when $w = 0$, Eq.(10) is obtained:

$$[\bar{A}_{\xi 0} + F_1 K_c F_2 + F_3 F_4 + F_5 K_c F_2]^T P_1 + P_1 [\bar{A}_{\xi 0} + F_1 K_c F_2 + F_3 F_4 + F_5 K_c F_2] < 0 \quad (10)$$

If $\int_0^t (z^T z - \gamma^2 w^T w) dt < 0$ is wanted to be tenable when $w \neq 0$, only $z^T z - \gamma^2 w^T w + \dot{V}(\xi) < 0$ is needed.

According to reference [5] lemma 2.2, the established condition to the above equation is:

$$\begin{aligned} & [\bar{A}_{\xi 0} + F_1 K_c F_2 + F_3 F_4 + F_5 K_c F_2]^T P_1 + P_1 [\bar{A}_{\xi 0} + F_1 K_c F_2 + F_3 F_4 + F_5 K_c F_2] \\ & + C_\xi^T C_\xi + \frac{1}{\gamma^2} P_1 B_{\xi w}^T B_{\xi w} P_1 < 0 \end{aligned} \quad (11)$$

Apply reference [5] lemma 2.1: there exists $\alpha > 0$ which brings Eq.(12) into existence:

$$\begin{aligned} & \bar{A}_{30}^T P_1 + P_1 \bar{A}_{30} + C_\xi^T C_\xi + \frac{1}{\gamma^2} P_1 B_{3w}^T B_{3w} P_1 + \alpha P_1 F_3 F_3^T P_1 + \frac{1}{\alpha} F_6^T F_6 \\ & + F_2^T K_c^T (F_1 + F_3)^T P_1 + P_1 (F_1 + F_3) K_c F_2 < 0 \end{aligned} \quad (12)$$

In Eq.(12): $F_6 = [F_\tau, 0, 0, 0]_{1 \times 6}$; $F_\tau = [\tau_{m1}, \tau_{m2}, \tau_{m3}]$. If there exists scalar σ to make Eq. (13) tenable by the applying of Finsler lemma^[6]:

$$\begin{aligned} & P_1^T \bar{A}_{30}^T + \bar{A}_{30} P_1 + P_1^T C_\xi^T C_\xi P_1 + \frac{1}{\gamma^2} B_{3w}^T B_{3w} + \alpha F_3 F_3^T + \frac{1}{\alpha} P_1^T F_6^T F_6 P_1 \\ & - \sigma (F_1 + F_3)(F_1 + F_3)^T < 0 \end{aligned} \quad (13)$$

$K_c = -\frac{\sigma}{2}(F_1 + F_3)^T P_1 F_2$ is got.

$(F_1 + F_3)(F_1 + F_3)^T \geq F_7 F_7^T$ and $F_7 = \begin{bmatrix} 0 & (1-a_1)B_2 \\ I & 0 \end{bmatrix}$, $a_1 = (1+a_1)(1+a_2)(1+a_3) - 1$. With the applying of Schur complement lemma^[4], Eq. (13) could be expressed as linear matrix inequality :

$$\begin{bmatrix} P_1^T \bar{A}_{30}^T + \bar{A}_{30} P_1 + \alpha F_3 F_3^T - \sigma F_7 F_7^T & B_{3w} & P_1^T C_\xi^T & P_1^T F_6^T \\ B_{3w}^T & -\gamma^2 I & 0 & 0 \\ C_\xi^T P_1 & 0 & -I & 0 \\ F_6 P_1 & 0 & 0 & -\alpha I \end{bmatrix} < 0 \quad (14)$$

Take $P_1^{-1} = X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12} & X_{13} \end{bmatrix}$ and take the matrices into Eq.(14), Eq.(15) is obtained:

$$\begin{bmatrix} \Sigma_1 & AX_{12} & \hat{B}_1 & X_{11} C_2^T & X_{11} F_\tau^T \\ X_{12} A^T & -\sigma I & 0 & X_{12} C_2^T & X_{12} F_\tau^T \\ \hat{B}_1^T & 0 & -\gamma^2 I & 0 & 0 \\ C_2 X_{11} & C_2 X_{12} & 0 & -I & 0 \\ F_\tau X_{11} & F_\tau X_{12} & 0 & 0 & -\alpha I \end{bmatrix} < 0 \quad (15)$$

$\Sigma_1 = AX_{11} + X_{11} A^T + \alpha B_2 B_2^T - \sigma(1-a_1) B_2 B_2^T$ in Eq.(15). Eq.(15) is solved to get P_1^{-1} , and then $K_c = -\frac{\sigma}{2} F_7^T P_1 F_2$ is got. So far, the system's controller is obtained.

4. The Design to the Anti-windup Compensator

Bring Eq.(8) into Eq.(7), Eq.(16) is got:

$$\begin{cases} \dot{\xi} = A_\xi \xi + B_q q + B_{3w} w \\ v = C_v \xi \\ z = C_z \xi \end{cases} \quad (16)$$

In Eq. (16): $q = v - \alpha q^T(v)$, $B_q = B_\theta + B_{\theta} E_c + \Delta B_\theta$, $B_\theta = \begin{bmatrix} -B_2 \\ 0 \end{bmatrix}$, $B_{\theta} = \begin{bmatrix} 0 \\ I \end{bmatrix}$, $\Delta B_\theta = \begin{bmatrix} -\Delta B_2 \\ 0 \end{bmatrix}$, $\Delta B_q \Delta B_q^T \leq \begin{bmatrix} \alpha_4^2 B_2 B_2^T & 0 \\ 0 & 0 \end{bmatrix} = F_8 F_8^T$, $F_8 = \begin{bmatrix} \alpha_4 B_2 \\ 0 \end{bmatrix}$, $C_v = (D_c \quad C_c)$

If there exists Lyapunov function $V_1(\xi) > 0, \dot{V}_1(\xi) < 0$ which makes $\int_0^t (z^T z - \gamma_1^2 w^T w) < 0$, Eq. (17) is stable and L_2 gain is smaller than γ_1 . Among them $V_1(\xi) = \xi^T P_\xi \xi$, P_ξ is the positive definite symmetric matrix. Derivative \dot{V}_1 we can get that:

$$\begin{aligned} \dot{V}_1(\xi) &= \xi^T (\bar{A}_\xi P_\xi + P_\xi \bar{A}_\xi) \xi + 2\xi^T P_\xi B_q q + 2\xi^T P_\xi B_{3w} w \\ &\leq \xi^T \left(\bar{A}_\xi P_\xi + P_\xi \bar{A}_\xi + \alpha_2 P_\xi F_3 F_3^T P_\xi + \frac{1}{\alpha_2} F_6^T F_6 + \alpha_3 P_\xi F_8 F_8^T P_\xi + \frac{1}{\alpha_3} F_2^T K_c^T K_c F_2 \right) \xi + 2\xi^T P_\xi B_\theta q + 2\xi^T P_\xi B_{\theta} E_c q + \frac{1}{\alpha_4} \xi^T P_\xi \Delta B_\theta \Delta B_\theta^T P_\xi \xi + \alpha_4 q^T q + 2\xi^T P_\xi B_{3w} w \\ &\leq \xi^T \left[\bar{A}_\xi P_\xi + P_\xi \bar{A}_\xi + P_\xi \left(\alpha_2 F_3 F_3^T + \alpha_3 F_8 F_8^T + \frac{1}{\alpha_4} F_6 F_6^T \right) P_\xi + \frac{1}{\alpha_2} F_6^T F_6 + \frac{1}{\alpha_3} F_2^T K_c^T K_c F_2 \right] \xi + 2\xi^T P_\xi B_\theta q + 2\xi^T P_\xi B_{\theta} E_c q + 2\xi^T P_\xi B_{3w} w + \alpha_4 q^T q \end{aligned} \quad (17)$$

Among them, $\alpha_2, \alpha_3, \alpha_4, \gamma_1$ are positive numbers. Notice that positive number α_5 could make $\alpha_5 q^T(v-q) = \alpha_5 q^T(C_v \xi - q) \geq 0$ tenable. If:

$$\dot{V}_1(\xi) + z^T z - \gamma_1^2 w^T w + 2\alpha_5 q^T(v-q) < 0 \quad (18)$$

The system could be stable.

5. Simulation Study

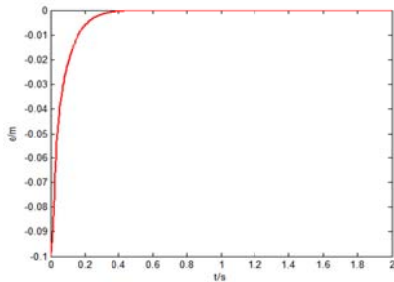


Fig. 2 Tracking error e

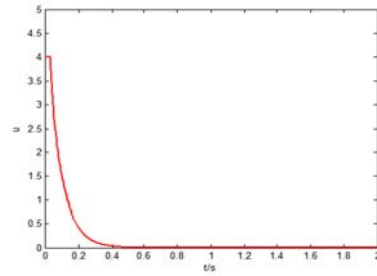


Fig. 3 Controller u

The simulating of the designed controller is shown in Fig.2 and Fig.3. When the step signal is followed, the implying of the anti-windup controller could be faster in eliminating of the tracking error, withdrawing from the saturated zone and in the stabilization of the system.

6. Conclusions

The results indicate the anti-windup controller in the present design could track the input signal in a high speed, and have good robustness and adaptability for the parameter uncertainty.

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