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An improved method of MRI segmentation based on variational level set

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Abstract: Intensity inhomogeneity is a thorny problem in MRI segmentation. In order to solve this problem, an improved method of MRI segmentation is proposed in this paper. The polarity information is introduced to solve the problem and an energy penalty term is introduced to make sure that the level set function keep approaching the symbol distance function. In the method of this paper, the problem of image segmentation is attributed to a problem of minimum of the energy function with local polar information. The improved symbol distance function is built firstly. Then, the final segmentation result is got through solving the minimum value of the energy function by variational level set. Proved by lots of experiments, this method is very efficient.

1. Introduction

Image segmentation is the basis of image analysis. The methods on image segmentation can be divided into two categories: methods based on edges and methods based on regions[1,2]. The former depends mainly on the gradient information. These methods are not ideal to the images with weak boundary. The latter mainly dependents on region information: intensity, color or texture. These methods can deal with complex images. But, the time is expensive. In recent years,some methods based on energy function minimization were proposed[3]. The thoughts of these methods consider image segmentation as a minimization problem of energy function. When the energy function reaches its minimum value, the image segmentation problem is solved. The method with level set was proposed by Osher and Sethian in 1988[4]. The variational level set method was proposed by Cai[5]. In order to avoid the question of re-initialization, Zhang proposed an improved variational level set method by adding a penalty term into the energy function[6]. Chan and Vese proposed the C-V model which was more efficient than variational level set method. C-V model is based on an assumption that the intensity of foreground and background is homogeneity. But, this method is not ideal in dealing inhomogeneity images. And, re-initialization is also a flaw of this method. Li proposed a model DRLSE with a constraint term which can solve the problem of re-initialization[7].

In this paper, an improved variational level set method of MRI segmentation based on variational level set is proposed. The method is mainly used in medical images. The local polar information is used instead of the gradient information. The local polar information measures the direction of gradient vector near pixels, which can increase the pixel affection to edge. On the other hand, the symbol distance function is improved in this paper which can avoid the re-initialization. Lots of experiments proved that the method in this paper shows higher accuracy and lower computational complexity.

2. The relevant theories

2.1 Level set

The thought of active contour model is to attribute the segmentation to minimizing energy function. When the energy function reaches its minimum by variational level set, the curve is just the segmentation curve. The evolutionary curve is C(p,t). The initiative contour curve is $C_0(p)$. C(p,t) is the zero level set function of high dimension function, z=Φ(x,y,t). Z is the shortest distance of P(x,y) to the curve. Generally, if the function symbol is negative, the point is inside the evolutionary curve. Otherwise, the point is outside the evolutionary curve. The curve evolutiona equation expressed with level set is as follow,

$$\frac{\partial \emptyset}{\partial t} = -\nabla\emptyset \cdot \mathrm{FN} = -\nabla\emptyset \cdot \mathrm{F}\left(\frac{\nabla\emptyset}{|\nabla\emptyset|}\right) = \mathrm{F}|\nabla\emptyset| \tag{1}$$

F is the rate of evolution curve along the normal.

2.2 Improved symbol distance function

The active contour model demands that the level set function is just the symbol distance function, $|\nabla\emptyset|$=1. Otherwise, the iteration will be unsteady. Generally, the level set should be re-initialized after several iterations. The formula to initialize the level set is as follow,

$$\frac{\partial \emptyset}{\partial t} = \mathrm{sing}(\emptyset)(1 - |\nabla\emptyset|) \tag{2}$$

Φ is the level set function. When evolution is steady, $|\nabla\emptyset|$=1. The initialized level set function is used as the new initiative level set function. So, the initialization of level set function will cost lots of times. Document [5] propose a method to void initialization by introducing a constraint of symbol distance function,

$$\mathrm{P}(\emptyset) = \frac{1}{2}\int(|\nabla\emptyset(\mathrm{p}) - 1|)^2 \mathrm{dx} \tag{3}$$

The function value is computed by the content that the level set deviates the symbol distance function. The computation function of differential rate is as follow,

$$\mathrm{d_p(s)} = \frac{\mathrm{p}'}{\mathrm{s}} \tag{4}$$

In the formula, s=$\nabla\emptyset$. The differential rate is as follow,

$$\mathrm{d_p} = 1 - \frac{1}{\nabla\emptyset} \tag{5}$$

When $|\nabla\emptyset|\geq1$, $\mathrm{d_p}(|\nabla\emptyset|)$ is positive and the difference is forward and $\nabla\emptyset$ decreases. When $|\nabla\emptyset|\leq1$, $\mathrm{d_p}(|\nabla\emptyset|)$ is negative and the difference is backward and $\nabla\emptyset$ increases. When $|\nabla\emptyset|$ approaches 0, the difference rate is infinite which will cause the symbol distance function shocking near 0. So, Li proposed a Double-well symbol distance function,

$$\mathrm{P_2}(|\nabla\emptyset| = \begin{cases} \frac{1}{(2\pi)^2}(1 - \cos(2\pi|\nabla\emptyset|), & \text{if } |\nabla\emptyset| \leq 1 \\ \frac{1}{2}(1 - |\nabla\emptyset|)^2, & \text{if } |\nabla\emptyset| \geq 1 \end{cases} \tag{6}$$

This function is used to compute the function value when level set deviates symbol distance function. This function will get its minimum value at position 0 and 1. The difference rate can be computed by formula (7),

$$\begin{cases} d_{p2} = \frac{\sin(2\pi|\nabla\emptyset|)}{2\pi|\nabla\emptyset|}, |\nabla\emptyset| \leq 1 \\ d_{p2} = 1 - \frac{1}{|\nabla\emptyset|}, |\nabla\emptyset| \geq 1 \end{cases} \tag{7}$$

When $|\nabla\emptyset| \geq 1$, the difference is positive and forward. $|\nabla\emptyset|$ decreases. When $0.5 \leq |\nabla\emptyset| \leq 1$, $d_{p2}(|\nabla\emptyset|)$ is negative and backward. $|\nabla\emptyset|$ increases. When $|\nabla\emptyset| \leq 0.5$, $d_{p2}(|\nabla\emptyset|)$ is positive and forward. The $|\nabla\emptyset|$ approaches 0. So, the improved difference is finite in the whole field. So, the shocking of the symbol distance function can be avoided.

2.3 Polarity information

The polar value of pixel can distinguish the location of pixel accurately. Compare to gradient information, the polar information is more accurate. Polar information is a local characteristic. The expression of polar value is as follow,

$$p(x) = \frac{|E_+ - E_-|}{E_+ + E_-} \tag{8}$$

$$\begin{cases} E_+ = \sum_{u,v} G_\sigma(u,v)[\nabla I \cdot \vec{n}]_+ \\ E_- = \sum_{u,v} G_\sigma(u,v)[\nabla I \cdot \vec{n}]_- \end{cases} \tag{9}$$

In this formula, σ is gauss window function of smooth Gaussian kernel with width σ. $[]_+$ and $[\nabla I \cdot \vec{n}]_-$ is the parameter adjusting of positive and negative. E_+ shows the number of gradient vector of positive in the window function. E_- shows the number of gradient vector of negative in the window function. The polar value is between 0 and 1. When the polar value approaches 1, the pixel is just on the edge.

3. Our segmentation model

The active contour is mainly used to segment the object in an image. The equation is consists of two parts: internal force and external energy. The internal energy make the curve to move toward inside. The external energy ensures the curve to stop when reach the edge of object. Image segmentation is completed when evolution curve reaches the edge by minimizing energy function.

So, the question of re-initiation can be avoided by the improved symbol distance function. The computation is steady and the complexity is lower. The active contour model based on local polar information can well deal with the inhomogeneity image. So, the polar information is introduced as the local intensity. In this paper, the advantages of polarity information and improved symbol distance function are combined together. The improved symbol distance function is used as the internal energy of energy function and the local polar information is used as external energy. The energy function in this paper is shown as follow,

$$E(\emptyset) = \mu p_2(\emptyset) + \lambda \int_\Omega g_p\, \delta(\emptyset)|\nabla\emptyset| dxdy + \alpha \int_\Omega g_p\, H(\emptyset) dxdy \tag{10}$$

E is the whole energy function. u, λ,$\alpha$are all constant more than 0. $H(\emptyset)$ is Heaviside function. $\delta(\emptyset)$ is Dirac function. $\emptyset$ is level set function. The expressions of Heaviside function and Dirac function after regularization are shown as follow,

$$\begin{cases} H_{\varepsilon}(\emptyset) = \frac{1}{2}\left(1 + \frac{2}{\pi}\right)\arctan\left(\frac{\emptyset}{\varepsilon}\right) \\ \delta_{\varepsilon}(\emptyset) = \frac{1}{\pi}\frac{\varepsilon}{\varepsilon^2 + \emptyset^2} \end{cases} \tag{11}$$

g_p is a stop condition of evolutionary curve which expression is shown as follow,

$$g_p = 1 - P(I) \tag{12}$$

$P(I)$ is the polar value of pixel. The evolution equation of level set function can be gotten by variational level set.

$$\frac{\partial \emptyset}{\partial t} = \mu \mathrm{div}(d_{p2}|\nabla\emptyset|\nabla\emptyset) + \lambda\delta_{\varepsilon}(\emptyset)\mathrm{div}\left(g_p\frac{\nabla\emptyset}{|\nabla\emptyset|}\right) + \alpha g_p \delta_{\varepsilon}(\emptyset) \tag{13}$$

The first item at right is a constraint item. The last two items at right control the segmentation edge.

4. Experiment and analysis

In the experiments, comparisons are carried at this method in this paper, DRLSE and CV.

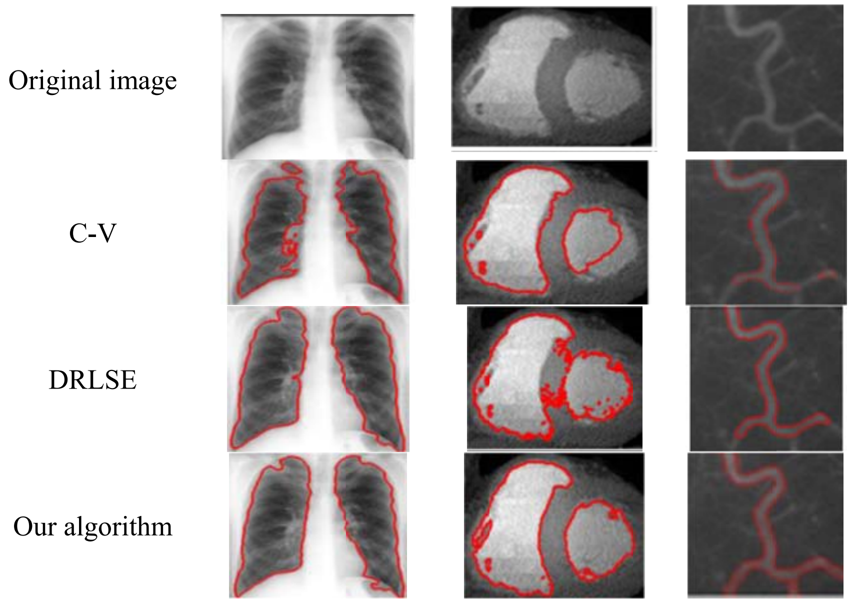

Figure 1 experiment results

The experiment conditions: Windows 8, I5 CPU, 4GB, Mat lab 2016a. The parameters are as following: α=1.6, μ=0.05, λ=5.The method proposed in this paper is mainly applied in MRI segmentation. These images are all selected from two databases about MRI, Caltech101and Caltech256. To verity the validity of the method in this paper, these picture selected form these database are all noisy and inhomogeneous, and the segmentation results of this paper, DRLSE and CV are shown as Fig. 1.

The experiments are carried out at the same conditions, including hardware and software. Seen from the experiments, CV and DRLSE can't segment these images correctly and some errors occur in some segmentation. The method in this paper can do well. So, we can see that the method in this paper is efficient and robust. The method is very efficient in deal with inhomogeneous images.

Table 1 times comparisons

	Image 1	Image 2	Image 3
C-V	4.42	3.74	4.11
DRLSE	4.81	4.12	4.36
Our method	4.50	3.81	4.23

Seen from table 1, CV cost less times, but the effect is lower which can't satisfy the demand of application. The amount of time of the method in this paper is lower comparing to DRLSE.

So, the method in this paper is efficient and robust, especially in dealing with inhomogeneous images.

5. Conclusions

In order to solve the problem of inhomogeneous intensity segmentation, an improved method of MRI segmentation based on vibrational level set is proposed combining polar information and DLSE method. In this method, polar information and improved symbol distance function are introduced into the active contour model. The energy function of evolution curve is built by variational level set. The polar information is steady. The improved symbol distance function can void re-initialization. So, the method is steady and efficient.

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