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The Optimal Combination Forecasting Based on ARIMA, VAR and SSM

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Abstract: In order to overcome the defects of a single time series forecasting model and to improve the prediction accuracy, this paper proposes an improved optimal combination model with Artificial Bee Colony algorithm used to solve the optimal weight coefficient automatically. Taking the Manufacturers' Shipments as an example to analyze, we use ARIMA、VAR and SSM to forecast the shipments respectively. Based on these three models, we construct the optimal combination forecasting model. By inspection, it is superior to the other three models in accuracy.

1. Introduction

One of the purpose of time series analysis is to forecast, which means predicting the future values of a series based on the history of that series and, possibly, other related series of factors [1] .It shows that no single method can be applied to the prediction of all time series. In order to overcome the limitation of the single forecasting model, people combine various forecasting models and use the comprehensive information given by them in proper manner to get the final forecasting result.

The manufacturing industry reflects a country's productivity level, playing a significant role in the economic development of a country. More accurate forecasting of the Manufacturers' Shipments is very important. In this paper, three time series models of ARIMA、 VAR and SSM were used to forecast respectively first. Then based on these three models, we automatically get the optimal weights via Artificial Bee Colony algorithm so as to construct the optimal combination model to forecast.

2. Review of related methodologies

2.1 ARIMA model

The single integrated autoregressive moving average (ARIMA) model which is also known as Box-Jenkins model because of its simplicity, feasibility and flexibility.

If the transferred series is stationary, we can use ARIMA model to forecast. For modeling of seasonal time series beside non-seasonal series, $ARIMA(p,d,q)(P,D,Q)s$ known as multiplicative ARIMA model is defined as follows:

$$\begin{aligned}&(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)(1-\Phi_1 B^s-\Phi_2 B^{2s}-\cdots-\Phi_P B^{Ps})(1-B)^d(1-B^s)^D X_t \\ &=(1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q)(1-\Theta_1 B^s-\Theta_2 B^{2s}-\cdots-\Theta_Q B^{Qs})\omega_t\end{aligned} \quad (1)$$

where ω_t is the random variable, s is the periodic term, B is the difference operator as $B(X_t) = X_{t-1}$, $(1-B^s)^D$ is the Dth seasonal difference measure s, $(1-B)^d$ is the dth non-seasonal difference, p and q is the order of non-seasonal autoregressive model and moving average model respectively, P and Q is the order of seasonal autoregressive model and moving average model respectively, ϕ and θ is the parameter of non-seasonal autoregressive model and moving average model respectively, Φ and Θ is the parameter of seasonal autoregressive model and moving average model respectively [2].

2.2 VAR model

The vector autoregressive (VAR) model is not based on economic theory, whose basic idea is that in each equations of the model, the endogenous variables regress their lagged values so as to estimate the long-term dynamic relationship between variables [3] as follows:

$$X_t = A_1 X_{t-1} + \cdots + A_p X_{t-p} + \Gamma \mu_t + \omega_t, \tag{2}$$

where X_t and μ_t is the endogenous and exogenous variable vector respectively, ω_t is the random variable, p is the lagged rank, $A_1, \cdots, A_p$ and Γ are the coefficient matrix to be estimated.

2.3 SSM

State space model (SSM) is often used to estimate the time variable that can not be observed in econometrics. It establishes the relationship between the observable variables and the internal system, and can achieve the purpose of analysis and prediction by estimating the different state vectors.

The general linear normal state space model is defined as follows:

$$\alpha_{t+1} = F\alpha_t + K\zeta_t, \tag{3}$$

$$Y_t = H\alpha_t + \varepsilon_t, \tag{4}$$

where α_t is the variable that can not be observed, Y_t is the observed value, ζ_t and ε_t are the random variables, and the three system matrix of F, K and H determine the structure of the model. Eq. (3) is the state equation, describing the state space evolution of a stochastic dynamical system. Eq. (4) is the observation equation, showing that the m-dimensional measurement Y_t is subject to a linear transformation of the hidden state α_t and is further corrupted by a measurement noise process ε_t.

3. Combination Forecasting Model

The core issue of combining forecasting is how to distribute the weight of each single prediction model, in order to improve the prediction accuracy effectively. Common methods are arithmetical average method, variance reciprocal method [4], mean square reciprocal method [5], standard deviation method, etc.

3.1 The Optimal Combination Forecasting model

This paper uses the optimal weighted method to determine the weight, whose basic idea is to construct the objective function of prediction error according to certain rules, and to determine the optimal weight in the combination forecasting model by solving the optimal value of the objective function under certain constraints.

In a certain prediction problem, we set x is the observation object and there are k kinds of methods for forecasting it: $\{x_1, x_2, \cdots, x_k\}$. The combination forecasting model can be represented as follows:

$$x(t)=\sum_{i=1}^{k}\omega_i(t)x_i(t) \qquad (t=1,2,\cdots,n) \tag{5}$$

where $x(t)$ and $x_i(t)$ is the predicted value of the i single model and the combination forecasting model in moment of t respectively, $\omega_i(t)$ is the weight of the i single in moment of t and meets

$$\sum_{i=1}^{k}\omega_i(t)=1,\ \omega_i(t)\geq 0 \quad (t=1,2,\cdots,n) \tag{6}$$

On the principle of minimizing the absolute value of the combined prediction error of the sample points, the mathematical expression of optimal combination forecasting model is as follows:

$$\begin{cases} \min J_t = |e_t| = |x(t)-\hat{x}(t)| = \left|\sum_{i=1}^{k}\omega_i(t)e_i(t)\right| \\ \quad s.t. \sum_{i=1}^{k}\omega_i(t)=1, \omega_i(t)\geq 0 \end{cases} \tag{7}$$

where $e_i(t)$ and $e(t)$ is the predicted error of the i single forecasting model and the combination forecasting model in moment of t respectively, $\hat{x}(t)$ is the actual value in moment of t.

3.2 Solving optimal weights by ABC algorithm

The solution to the model above is a constrained optimization problem, the Artificial Bee Colony (ABC) algorithm is used to solve the optimal problem on accounts of its high parallelism, randomness, self-adaptability and ease of implementation. The main steps of ABC algorithm are given below [6]:

Send the scouts onto the initial food sources
REPEAT
  Send the employed bees onto the food sources and determine their nectar amounts
  Calculate the probability value of the sources with which they are preferred by the onlooker bees
  Send the onlooker bees onto the food sources and determine their nectar amounts
  Stop the exploitation process of the sources exhausted by the bees
  Send the scouts into the search area for discovering new food sources,randomly
  Memorize the best food source found so far
UNTIL(requirements are met)

4. Simulation

In order to verify prediction performance of the combination forecasting model, we commit simulation with the model. The raw data, which comes from the U.S. Department of Commerce Web site, are used in the simulation. The data include the Manufacturers' Shipments, New Orders and Total Inventory from February 1992 to June 2015. The data set is divided into training set and test set, in which the training set data is from February 1992 to March 2015,while the test set data is from April 2015 to June 2015. By predicting the test set and calculating the prediction error of each single model, we use the ABC algorithm to solve the optimal weight according to Eq. (7) so as to establish the combination forecasting model on the principle of minimizing the absolute value of the combined prediction error of the sample points. The optimal combination weight obtained by using ABC algorithm is (0.1824 0.5640 0.2536).To test prediction effect, we introduce the following error indicators.

The mean absolute percent error (MAPE),

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|e_t|}{\hat{x}(t)} = \frac{1}{n}\sum_{i=1}^{n}\frac{|x(t)-\hat{x}(t)|}{\hat{x}(t)} \tag{8}$$

and the root mean square error (RMSE),

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[x(t)-\hat{x}(t)]^2} \tag{9}$$

where $x(t)$ and $\hat{x}(t)$ is predicted value and actual value in moment of t respectively, n is sample number. The comparison of forecasting results by different models is showed in the Tab.1.

From the Tab. 1, we can see that SSM achieved the highest precision among the three single model, but the optimal combination forecasting model can further raise the accuracy of simulation, i.e. the RMSE has declined by 449.3, and the MAPE has declined by 0.153 percent, which means the prediction accuracy is improved by 90 percent. So the proposed model has successfully realized the mutual supplement with each other in terms of advantages of ARIMA、VAR and SSM.

Table 1 Comparison of forecasting results by the four models

error analysis	ARIMA	VAR	SSM	the optimal combination forecasting model
MAPE	0.37%	0.21%	0.17%	0.017%
RMSE	2158.1	1235.8	541.2	91.9

5. Conclusions

This paper presents an improved optimal combination forecasting model based on ARIMA, VAR and SSM. When determining the weights of the model, the Artificial Bee Colony algorithm is used to solve the optimal weight coefficient automatically. Taking the Manufacturers' Shipments as an example to carry on the empirical analysis, we use ARIMA、VAR and SSM to forecast the shipments respectively. Based on these three models, we construct the optimal combination model to forecast. By inspection, the combination model established in this paper is superior to the other three single models. It can get better results in MAPE and RMSE, improving the prediction accuracy and stability of the model. So the proposed model is effective and has certain practical value.

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